

# Online Appendix to “Decomposing Firm Value”

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This online appendix reports the results from additional analyses and robustness checks.

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# 1 The Aggregation Bias in BXZ/LWZ and Alternative Estimation Procedure

Equation (21) in the main draft establishes an exact relationship between a firm’s observed valuation ratio and the model-implied valuation ratio. As explained in the draft, we perform the estimation at the portfolio-level as in Belo, Xue, and Zhang (2013) (henceforth BXZ), which in turn follow the original approach in Liu, Whited, and Zhang (2009) (henceforth LWZ). Unlike LWZ/BXZ, however, we estimate the model parameters by targeting cross-sectional portfolio-level moments that do not require aggregating the data to construct a portfolio-level aggregate valuation ratio. Here, we discuss in more detail why we modify the estimation procedure relative to BXZ/LWZ, and show that our estimation procedure allow us to recover the fundamental firm-level structural parameters.

**Aggregation in LWZ/BXZ** To understand the need for our estimation method, it is useful to revisit the aggregation procedure in LWZ/BXZ.<sup>1</sup> Following the approach in LWZ/BXZ, one would estimate the valuation equation at the portfolio-level by first computing the portfolio-level characteristics (e.g., the portfolio-level investment rates), and then plugging these characteristics directly in the valuation equation (21) to obtain the observed and the model-implied valuation ratios. Specifically, in year  $t$ , the portfolio  $j$  investment rate in physical capital is computed as

$$\frac{I_{jt}^K}{K_{jt}} = \frac{\sum_i I_{j,i,t}^K}{\sum_i K_{j,i,t}}, i \in \text{Portfolio } j \quad (1)$$

which is then substituted in equation (17) to obtain the portfolio-level shadow price of the physical capital stock. Similarly, the portfolio level observed valuation ratio and capital stocks are given by

$$VR_{jt} = \frac{\sum_i (P_{it} + B_{it+1})}{\sum_i A_{it}}$$

$$K_{jt} = \sum_i K_{j,i,t}, i \in \text{Portfolio } j.$$

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<sup>1</sup>Liu, Whited, and Zhang (2009) estimate the model predicted investment returns rather than valuation ratios using portfolio-level aggregated data. The two are closely related because, to a first order approximation, the investment return is the valuation equation in first differences.

The estimation would then proceed to estimate the parameter values by the Generalized Method  
of Moments (GMM) under the identification assumption that the model errors, computed as the  
20 difference between the portfolio-level aggregated observed and model-implied valuation ratios, are  
on average zero.

The LWZ/BXZ approach provides a powerful framework for identifying robust links between  
valuation ratios/stock returns and portfolio-level characteristics. In addition, this approach averages  
25 out the noise in firm-level data in a convenient and elegant manner. Unfortunately, the aggregation  
procedure in the LWZ/BXZ approach complicates the interpretation of the parameter estimates  
as we will show with model-implied data below, because it is subject to an aggregation bias.  
Specifically, by using the portfolio-level characteristics computed as in equation (1) to construct  
the shadow price of the capital input in equations (17) of the main draft, the procedure does not  
30 guarantee the recovery of the true firm-level structural parameters because the shadow prices of the  
capital inputs are, in general, nonlinear functions of the firm characteristics.

**Our Alternative Estimation Procedure** To recover the firm-level structural parameters we  
thus modify the econometric approach proposed in LWZ/BXZ. As noted, in theory, any moment  
of the observed firm-level valuation ratios in equation (21) should be equal to any corresponding  
35 moments of the model-implied firm-level valuation ratios. Thus, we target cross-sectional portfolio-  
level moments that do not require aggregating the data to construct a portfolio-level aggregate  
valuation ratio, hence avoiding the aggregation bias. Specifically, in each year, we compute the  
portfolio-level valuation ratio by taking the cross-sectional equal-weighted mean of the firm-level  
observed and model-implied valuation ratios, which we refer to as cross-sectional mean estimation.

40 We perform the estimation of the valuation equation (17) under the standard assumption that  
the portfolio-level valuation ratio moments are observed with error by the econometrician:

$$\overline{VR}_{jt} = \widehat{VR}_{jt}(\Theta) + \varepsilon_{jt}, \quad (2)$$

where  $\widehat{VR}_{jt}(\Theta)$  denotes the model-implied portfolio-level moment of the cross-section of firm-level  
valuation ratios for the firms in portfolio  $j$  at time  $t$ ,  $\Theta$  represents the vector of structural parameters,  
i.e.  $\Theta = [\theta_P, \theta_L, \theta_K, \theta_B]$ , and  $\varepsilon$  captures the error in the portfolio-level moments. Based on equation

45 (2), we estimate the model parameters by least squares (LS), that is, we minimize the distance between the portfolio-level observed and model-implied valuation ratios moments:

$$\hat{\Theta} = \arg \min_{\Theta} \frac{1}{TN} \sum_{t=1}^T \sum_{j=1}^N \left( \overline{VR}_{jt} - \widehat{VR}_{jt}(\Theta) \right)^2.$$

Thus, unlike LWZ and BXZ, who estimate the model parameters by matching the time series means of the observed and model-implied portfolio valuation ratios, the use of LS in our estimation requires the model to match the realized time series of the observed cross sectional moments of the valuation ratios as close as possible. While in the absence of noise in the data, the GMM estimation is able to recover the true firm level parameters if one uses our aggregation procedure, we find that the time series data provides additional power to pin down the parameters when we use noisy firm level data.

To show the aggregation bias in LWZ/BXZ explicitly and how our procedure avoids this bias, we consider a particular calibration of the adjustment costs function in the context of the one-physical-capital input model.<sup>2</sup> We then use artificial data to investigate the ability of the two estimation approaches to recover the underlying firm-level structural parameters. We document that the parameter estimates using the aggregation procedure in LWZ do not have a structural interpretation. In addition, we verify that our alternative portfolio-level estimation method proposed in the main text allow us to recover the firm-level structural parameters.

For simplicity, we consider the one-physical-capital input model. To proceed, we generate data from a model economy in which the assumptions of the baseline investment model hold (and hence the firm-level observed and predicted (model-implied) valuation ratios are equal). But instead of simulating data from a model economy, we use real data as follows. We construct the capital stock process for each firm by using the law of motion:

$$K_{it} = (1 - \delta)K_{it-1} + I_{it}. \tag{3}$$

We use the firm-level physical capital investment data for  $I_{it}$  and the initial capital stock of the

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<sup>2</sup>Belo, Deng, and Salomao 2019 provide a general analysis of the aggregation bias and other economic issues in the context of empirical tests of investment-based models.

firm to be  $K_0$  and assume a depreciation of 10%. To generate (artificial) price data in this economy, we use the valuation equation implied by the neoclassical model, that is:

$$VR_{it} = \left\{ 1 + (1 - \tau_t)\theta \frac{I_t}{K_{t-1}} \right\}, \quad (4)$$

where  $VR_{it} \equiv \frac{P_{it}}{K_{it}}$  in which  $P_{it}$  is the market value of equity. Thus, by construction, the observed  
70 and the model-implied valuation ratio are equal (that is, the assumptions of the model are satisfied).

The econometric exercise of interest here is then to investigate the extent to which the different estimation approaches allow us to recover the structural parameters, which in our case is the parameter  $\theta$ . To make the results more general, we consider two values of the slope adjustment cost parameters  $\theta = 10$  or  $40$ . The curvature is fixed at 2 (quadratic), as in the baseline specification  
75 of the model in the main text. Given these parameters, we can generate a time series of (artificial) valuation ratios in the model using equation (4).

To examine the role of the impact of portfolio-level aggregation of the firm characteristics using the LWZ procedure, we first create 10 and 50 portfolios sorted on lagged investment. As in LWZ, we construct the portfolio-level counterpart of the valuation ratio as follows. For each portfolio  
80  $j = 1, \dots, 10$ , or  $50$ , and in each period, we have:

$$VR_{jt} = \frac{\sum_i^N P_{it}}{\sum_i^N K_{it}}, \quad i \in \text{Portfolio } j \quad (5)$$

$$I_{jt}/K_{jt-1} = \frac{\sum_i^N I_{it}}{\sum_i^N K_{it-1}}. \quad (6)$$

To estimate the model parameters we construct the model-implied predicted valuation ratio  $\widehat{VR}_{jt}$  as:

$$\widehat{VR}_{jt} \equiv 1 + (1 - \tau_t)\hat{\theta} \frac{I_t}{K_{t-1}}$$

which uses the portfolio-level investment rate computed as in equation (6). Following LWZ, we estimate the model parameters ( $\theta$ ) by the Generalized Method of Moments (GMM) using the  
85 moment condition:

$$E \left[ VR_{jt} - \widehat{VR}_{jt} \right] = 0, \quad j = 1, \dots, 10 \text{ or } 50. \quad (7)$$

We use the identity matrix as the weighting matrix. We label this method as GMM-XS. For comparison with the estimation approach used here that matches the time series data (and to establish that the conclusions here only depends on aggregation issues, and not on the estimation method (GMM versus LS, or time-series versus cross-sectional moments, used), we also estimate  
90 the parameters by minimizing the sum of squared residuals. That is, let

$$\varepsilon_{jt} = VR_{jt} - \widehat{VR}_{jt}.$$

We then estimate the model parameters by minimizing the objective function:  $\sum_{t=1}^T \sum_{j=1}^N \varepsilon_{jt}^2$ . We denote this method as LS-TS. For each estimation method, we report the parameter estimate of the slope coefficient  $\theta$  (reported as  $\hat{\theta}$ ) for the two cases  $\theta = 10$ , or  $40$ , together with the estimation bias, computed as the percentage deviation of the estimated parameter value relative to the true  
95 parameter value (bias =  $\frac{\hat{\theta} - \theta}{\theta}$ ).

Table 1, rows LWZ/BXZ, report the estimation results using the LWZ/BXZ aggregation method, and rows Our-XSM, report the estimation results using the our cross-sectional equally weighted mean method. The first panel reports the results using the 10 investment-rate (IK) portfolios, and the second reports the results using 50 IK portfolios. The columns on the right report the results  
100 using the GMM-XS estimation approach (that is, matching the time series average of the cross section of the portfolios), while the columns on the left report the results using LS-TS estimation approach (that is, matching the time series realization of each portfolio).

Table 1 reveals that, across all cases, the parameter estimates using the LWZ/BXZ aggregation procedure differ from the true firm-level structural parameters, and hence do not have a structural  
105 interpretation. In all cases considered here, the bias in the estimation ranges from -16.40% to -1.40%, and is never zero. Also, the parameter estimates vary significantly across the number of portfolios (10 vs. 50) and across the estimation procedures (GMM-XS vs LS-TS), which should not occur in large samples if the estimation procedure is consistent, in which case the procedure should recover the true underlying parameter values. Indeed, the variation of the parameter estimates across test  
110 assets helps us understand why the parameter estimates reported in LWZ vary significantly across different test assets used in the estimation. The bias occurs because of the aggregation issues in the

**Table 1: Comparison of Estimation Methods: the Impact of Portfolio-Level Aggregation**

This table reports the estimates of the model parameters across different portfolio-level aggregation methods for the one-physical-capital input model with curvature equal to 2, and the slope adjustment cost parameter represented by  $\theta$ . We consider two values of true model parameters at the firm level:  $\theta = 10$  or  $\theta = 40$ . For each method,  $\hat{\theta}$  is the estimated parameter, and bias is the percentage deviation of the estimated parameter value relative to the true parameter value (bias =  $\frac{\hat{\theta} - \theta}{\theta}$ ). In LWZ/BXZ the data is aggregated by first aggregating the firm characteristics to obtain the portfolio-level predicted valuation ratio as described in this appendix. XSM is the equal-weighted cross sectional mean aggregation method in which we compute the portfolio-level observed and predicted cross sectional valuation ratio across all the firms in the portfolios in each year. The test assets are 10 and 50 investment rate portfolios. Two estimation methods are used. In LS-TS the parameters are obtained by minimizing the sum of squared portfolio-level residual (the difference between observed and model-implied valuation ratio) at the portfolio-level. In GMM-XS the parameters are obtained by matching the average observed and predicted valuation ratio of each portfolio (as in LWZ/BXZ).

True Value:	LS-TS				GMM-XS			
	$\theta = 10$		$\theta = 40$		$\theta = 10$		$\theta = 40$	
Estimate:	$\hat{\theta}$	Bias (%)	$\hat{\theta}$	Bias (%)	$\hat{\theta}$	Bias (%)	$\hat{\theta}$	Bias (%)
	Estimation using 10 IK Portfolios							
LWZ/BXZ	8.72	-12.80	34.91	-12.73	8.36	-16.40	33.44	-16.40
Our-XSM	10.00	0.00	40.00	0.00	10.00	0.00	40.00	0.00
	Estimation using 50 IK Portfolios							
LWZ/BXZ	9.86	-1.40	39.47	-1.32	9.73	-2.70	38.91	-2.73
Our-XSM	10.00	0.00	40.00	0.00	10.00	0.00	40.00	0.00



procedure. The non-linearities in the valuation ratio mean that the true portfolio-level valuation ratio is different from the portfolio-level valuation ratio obtained by first aggregating each portfolio-level characteristics (investment rate, etc.) separately, to construct the portfolio-level valuation  
115 ratio counterparts.

Turning to the analysis of the performance of the alternative estimation procedure proposed in the main text, namely the use of equal-weighted cross-sectional mean, Table 1 shows that this procedure avoids the aggregation issues in LWZ/BXZ. In particular, the results in Table 1 show that the alternative aggregation procedure is unbiased, thus allowing us to recover the true underlying  
120 firm-level structural parameters. We also have done tests with the multiple capital input model and find that, even with noise, the proposed estimation of the paper is able to recuperate the adjustment costs parameters.

Naturally, with measurement error, the analysis becomes significantly more complicated. Since measurement error in firm-level data is not directly observed, different assumptions about the  
125 nature of the error may lead to different results. This does not necessarily invalidate the previous analysis, however. The analysis here shows that, even without measurement error, the aggregation procedure in LWZ/BXZ contaminates the parameter estimates and hence prevents them from having a structural interpretation.

## 2 The Constant Returns to Scale Assumption of the Operating 130 Profit Function

Here we discuss the constant returns to scale (CRS) assumption of the operating profit function, and provide theoretical and empirical support for this specification. This assumption greatly simplifies the estimation of the model because it allows for a closed form solution of the equilibrium market value of the firm.

135 Subsection 2.1 shows that a specification of the operating profit function that is homogeneous of degree one (or, equivalently, CRS) in the capital inputs is consistent with a specification in which the firm's production technology exhibits decreasing returns to scale (DRS) in a subset of the inputs and the firm has market power (faces a downward sloping demand curve) and optimally chooses

output price to maximize profits. Subsection 2.2 uses applied economics methodology to estimate  
140 the parameters of the production function.

## 2.1 Interpretation: Decreasing Returns to Scale and Market Power

Assume the firm faces the following downward-sloping demand curve:

$$P_t = B_t^\gamma Q_t^{\frac{1}{\varepsilon}}, \gamma > 0 \text{ and } \varepsilon < -1 \quad (8)$$

where  $B_t$  is the firm's stock of brand capital (which affects consumer's willingness to pay (WTP) for the good),  $P_t$  is the price of the good (chosen by the firm) and  $Q_t$  is the quantity demanded. The  
145 parameters satisfy the constraints  $\gamma > 0$  (impact of brand capital on consumers' WTP is positive) and  $\varepsilon < -1$  is the demand elasticity, which is assumed to be less than minus one, that is, demand is elastic. We are ruling out an inelastic demand because it is never optimal for a monopolist to operate in the inelastic portion of the demand curve.

Now, assume the firm's production function is given by:

$$Q_t = X_t E_t^{\alpha_E}, \alpha_E > 0 \text{ and } \alpha_K + \alpha_E \leq 1 \quad (9)$$

150 where  $E_t$  captures the variable (non quasi-fixed) inputs (for example, energy or other materials). To save on notation, we bundled all the quasi-fixed capital inputs and the productivity level in the variable  $X_t$ . For example, if the only quasi-fixed input in the *production* function is physical capital, we have:

$$X_t = A_t K_t^{\alpha_K}, \alpha_K > 0 \text{ and } \alpha_K + \alpha_E \leq 1. \quad (10)$$

If we have labor or other inputs, we can just re-specify  $X_t$  accordingly.

155 We can write the operating profit function as a function of the firm's quasi-fixed capital inputs. This is obtained by choosing the optimal level of the variable inputs that maximize the per period

profits:

$$\begin{aligned} \max_{E_t} \Pi_t &= P_t Q_t - c_t E_t \\ &s.t. (8) \text{ and } (9) \end{aligned}$$

where  $c_t$  is the marginal cost of one unit of the variable inputs. Substituting equations (8) and (9) in the objective function we have

$$\max_{E_t} \Pi_t = B_t^\gamma X_t^{1+\frac{1}{\varepsilon}} E_t^{\alpha_E(1+\frac{1}{\varepsilon})} - c_t E_t \quad (11)$$

160 The first order condition (FOC) w.r.t. to  $E_t$  is:

$$\alpha_E \left(1 + \frac{1}{\varepsilon}\right) B_t^\gamma X_t^{1+\frac{1}{\varepsilon}} E_t^{\alpha_E(1+\frac{1}{\varepsilon})-1} = c_t \quad (12)$$

To facilitate the algebra, multiplying both sides of the FOC (12) by  $E_t$  and then re-arranging the terms we obtain:

$$\alpha_E \left(1 + \frac{1}{\varepsilon}\right) B_t^\gamma Q_t^{1+\frac{1}{\varepsilon}} = E_t^* c_t$$

Next, we substitute  $c_t E_t$  in the objective function (11) at the optimum, and obtain:

$$\Pi_t^* = \left(1 - \alpha_E \left(1 + \frac{1}{\varepsilon}\right)\right) B_t^\gamma Q_t^{1+\frac{1}{\varepsilon}} \quad (13)$$

Note that we need to find the optimal level of the variable inputs. We can solve equation (12) for

165  $E_t^*$

$$E_t^* = \left[ \frac{1}{\alpha_E \left(1 + \frac{1}{\varepsilon}\right)} B_t^{-\gamma} X_t^{-1-\frac{1}{\varepsilon}} c_t \right]^{\frac{1}{\alpha_E(1+\frac{1}{\varepsilon})-1}} \quad (14)$$

Now, substitute equation (14) into equation (13) in  $Q_t^* = X_t E_t^{*\alpha_E}$  to get:

$$\Pi_t^* = \tilde{A}_t B_t^{\gamma \left(1 - \frac{\alpha_E (1 + \frac{1}{\varepsilon})}{\alpha_E (1 + \frac{1}{\varepsilon}) - 1}\right)} K_t^{\alpha_K (1 + \frac{1}{\varepsilon}) \left(1 - \frac{\alpha_E (1 + \frac{1}{\varepsilon})}{\alpha_E (1 + \frac{1}{\varepsilon}) - 1}\right)} \quad (15)$$

where:

$$\tilde{A}_t \equiv \frac{(1 - \alpha_E (1 + \frac{1}{\varepsilon}))}{\frac{\alpha_E (1 + \frac{1}{\varepsilon})}{\alpha_E (1 + \frac{1}{\varepsilon}) - 1}} c_t^{\frac{\alpha_E (1 + \frac{1}{\varepsilon})}{\alpha_E (1 + \frac{1}{\varepsilon}) - 1}} A_t^{(1 + \frac{1}{\varepsilon}) \left(1 - \frac{\alpha_E (1 + \frac{1}{\varepsilon})}{\alpha_E (1 + \frac{1}{\varepsilon}) - 1}\right)}$$

which is always a positive number. We can interpret this variable as the *effective* productivity level of the firm in the operating profit function.

170 For the operating profit function to have CRS in the two quasi-fixed capital inputs (physical capital and brand capital,  $K_t$  and  $B_t$ , respectively) we need:

$$\underbrace{\gamma \left(1 - \frac{\alpha_E (1 + \frac{1}{\varepsilon})}{\alpha_E (1 + \frac{1}{\varepsilon}) - 1}\right)}_{\text{Coefficient on } B_t \text{ in } \Pi_t^*} + \underbrace{\alpha_K \left(1 + \frac{1}{\varepsilon}\right) \left(1 - \frac{\alpha_E (1 + \frac{1}{\varepsilon})}{\alpha_E (1 + \frac{1}{\varepsilon}) - 1}\right)}_{\text{Coefficient on } K_t \text{ in } \Pi_t^*} = 1 \quad (16)$$

so that the optimized operating profit function  $\Pi_t^*$  is CRS in the two capital inputs.

Now recall that  $\varepsilon < -1$  so that  $0 < 1 + \frac{1}{\varepsilon} < 1$ . Also,  $\gamma \geq 0$ ,  $\alpha_E + a_K \leq 1$  and  $\alpha_E > 0$  and  $\alpha_K > 0$ . All we need to do is to find a combination of parameters that work to prove that  
 175 the CRS specification can be consistent with a production function that is DRS and the firm has market power. Here, we list a series of numerical examples that illustrate the claim. For each case, we specify a set of plausible elasticity,  $\varepsilon$ , and share (in the production function,  $\alpha_K$  and  $\alpha_E$ ) parameters, and then solve for the required  $\gamma$  that solves equation (16) (and the corresponding coefficients on  $B$  and  $K$  in the optimized operating profit function):

180 Example #1:  $\varepsilon = -2$ ,  $\alpha_K = 0.3$ ,  $a_E = 0.3$ :

$$\gamma = 0.70 \text{ so that coef. on } B = 0.82 \text{ and coef. on } K = 0.18$$

Example #2:  $\varepsilon = -10$ ,  $\alpha_K = 0.3$ ,  $a_E = 0.3$ :

$$\gamma = 0.46 \text{ so that coef. on } B = 0.63 \text{ and coef. on } K = 0.37$$

Example #3:  $\varepsilon = -2$ ,  $\alpha_K = 0.7$ ,  $a_E = 0.1$ :

$$\gamma = 0.60 \text{ so that coef. on } B = 0.63 \text{ and coef. on } K = 0.37$$

Example #4:  $\varepsilon = -100$ ,  $\alpha_K = 0.5$ ,  $a_E = 0.1$ :

$$\gamma = 0.406 \text{ so that coef. on } B = 0.45 \text{ and coef. on } K = 0.55$$

Example #5:  $\varepsilon = -2$ ,  $\alpha_K = 0.7$ ,  $a_E = 0.3$ :

$$\gamma = 0.50 \text{ so that coef. on } B = 0.59 \text{ and coef. on } K = 0.41$$

185 All of these cases illustrate combinations of parameter values consistent with a case in which the specification of the firm's technology exhibits DRS in a subset of the inputs, the firm has market power, and the resulting operating profit function exhibits CRS.

## 2.2 Operating Profit Function Estimation

Obtaining consistent estimates of production/operating profit function parameters is challenging  
190 due to the simultaneity problem generated by the relationship between productivity and input demands. When subject to productivity shocks, firms respond by expanding their level of output and by demanding more inputs; negative shocks, on the other hand, lead to a decline in both output and demand for inputs. The positive correlation between the observable input levels and the unobservable productivity shocks is a source of bias in ordinary least squares (OLS) estimates. The  
195 applied economics literature has proposed the use of control function approaches to overcome this simultaneity problem. Olley and Pakes (1996) (henceforth OP) proposes a two-step approach that

uses investment data to proxy for productivity.<sup>3</sup> One downside of their approach is that they assume that labor inputs are non-dynamic and therefore inconsistent with costly adjustments. Akerberg, Caves, and Frazer (2015) (henceforth ACF) refines their approach and propose a methodology that  
200 allows labor to be costly to adjust.

Here, we follow the methodologies by Olley and Pakes (1996) and Akerberg, Caves, and Frazer (2015) to estimate the parameters of a Cobb-Douglas operating profit function using all four inputs: physical capital, labor, knowledge and brand capital.<sup>4</sup> We use both revenue (*sales* in Compustat) and value added (*sales-cogs* in Compustat).

205 Table 2 shows the parameter estimates using both methodologies for all firms together and for low and high skill firms separately. Overall, the coefficient estimates add up to a number very close to one (see last row, highlighted in bold). This result confirms that the CRS assumption of the operating profit function constitutes an empirically reasonable approximation.

### 3 Robustness Checks

210 To check the robustness of our main findings and, in particular, the importance of non-physical capital inputs for firm value, we re-estimate the model for different data samples and across several perturbations of the empirical procedures. First, we show how the results are impacted by assuming alternative adjustment costs functions that allow for asymmetric costs or a curvature parameter that is different from two (i.e., non-quadratic). Second, we show how the results change for different  
215 test assets, an alternative estimation method and across different data samples. Specifically, we estimate the model using: a larger number of portfolios, an alternative industry classification as portfolios, firm-level data (as opposed to performing the estimation using portfolios), the Euler equation approach using firm-level data, the sub-sample of firms that were excluded from the main sample due to missing (or always zero) R&D expenses data. Finally, we show the results from  
220 additional robustness checks (which includes tests using an alternative measure of intangible capital such as organization capital, following Eisfeldt and Papanikolaou 2013).

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<sup>3</sup>Levinsohn and Petrin (2003) is another influential approach. Their approach requires materials data that we do not observe.

<sup>4</sup>We use the stata module `prodest` which allows for both methodologies. See <https://ideas.repec.org/c/boc/bocode/s458239.html>.

Table 2: Parameter Estimates

This table shows production function parameter estimates using the methodologies of Olley and Pakes (1996) (OP) and Akerberg, Caves, and Frazer (2015) (ACF) for labor ( $L$ ), physical ( $K^P$ ), knowledge ( $K^K$ ) and brand ( $K^B$ ) capital. The estimates are based on sales and value added measures of production.

	All Firms						Low Skill						High Skill					
	sales		value-added		sales		value-added		sales		value-added		sales		value-added			
	OP	ACF	OP	ACF	OP	ACF	OP	ACF	OP	ACF	OP	ACF	OP	ACF	OP	ACF		
$L$	0.661*** (55.49)	0.662*** (170.53)	0.490*** (36.75)	0.585*** (10.71)	0.593*** (24.57)	0.623*** (48.24)	0.460*** (16.38)	0.376*** (5.06)	0.690*** (50.59)	0.690*** (110.54)	0.497*** (23.96)	0.404*** (16.90)						
$K^P$	0.134*** (4.43)	0.195*** (14.64)	0.144** (3.00)	0.123*** (5.19)	0.0820 (1.50)	0.143*** (3.36)	0.111** (2.03)	0.194*** (4.21)	0.119*** (4.63)	0.179*** (18.13)	0.141*** (3.13)	0.178*** (12.71)						
$K^K$	0.0403* (2.30)	0.0310** (2.69)	0.185 (1.92)	0.220*** (10.41)	0.0865 (1.50)	0.0736*** (2.71)	0.143 (1.62)	0.145*** (5.47)	0.0218 (0.65)	0.0505*** (3.66)	0.235** (2.44)	0.273*** (28.42)						
$K^B$	0.203*** (6.01)	0.151*** (13.22)	0.113 (0.97)	0.136*** (10.35)	0.170*** (2.93)	0.147*** (6.67)	0.179 (1.27)	0.265*** (9.29)	0.151*** (6.31)	0.130*** (10.98)	0.0725 (0.57)	0.167*** (20.20)						
<b>Sum</b>	<b>1.04</b>	<b>1.04</b>	<b>0.93</b>	<b>1.06</b>	<b>0.93</b>	<b>0.99</b>	<b>0.89</b>	<b>0.98</b>	<b>0.98</b>	<b>1.05</b>	<b>0.95</b>	<b>1.02</b>						

### 3.1 Alternative Adjustment Costs Functions

In the baseline model, we specify the adjustment costs function to be symmetric and quadratic. In this section, we relax these assumptions and show that they have a small impact on the model fit and on the conclusions from the model. In Section 3.1.1 we allow for asymmetric adjustment costs and in Section 3.1.2 we freely estimate the curvature of the adjustment costs function .

#### 3.1.1 Asymmetric Adjustment Costs

In the baseline model, we specify a symmetric adjustment costs function for parsimonious reasons and to avoid parameter proliferation. This assumption may be at odds with some results in the large investment and labor demand literature, however. For example, Abel and Eberly (1994) and Abel and Eberly (1996) show that allowing for asymmetry in physical capital adjustment costs (e.g., due to investment irreversibility) improves the ability of an otherwise standard neoclassical investment model to explain investment dynamics. Thus, here we consider a more flexible adjustment costs function where we allow the costs of adjusting each input to be different:

$$C_{it} = \frac{\theta_P}{v_P^2} \left[ \exp \left( -v_P \frac{I_{it}^P}{K_{it}^P} \right) + v_P \frac{I_{it}^P}{K_{it}^P} - 1 \right] K_{it}^P + \frac{\theta_L}{v_L^2} \left[ \exp \left( -v_L \frac{H_{it}}{L_{it}} \right) + v_L \frac{H_{it}}{L_{it}} - 1 \right] W_{it} L_{it} + \frac{\theta_K}{v_K^2} \left[ \exp \left( -v_P \frac{I_{it}^K}{K_{it}^K} \right) + v_P \frac{I_{it}^K}{K_{it}^K} - 1 \right] K_{it}^K + \frac{\theta_B}{v_B^2} \left[ \exp \left( -v_B \frac{I_{it}^B}{K_{it}^B} \right) + v_B \frac{I_{it}^B}{K_{it}^B} - 1 \right] K_{it}^B. \quad (17)$$

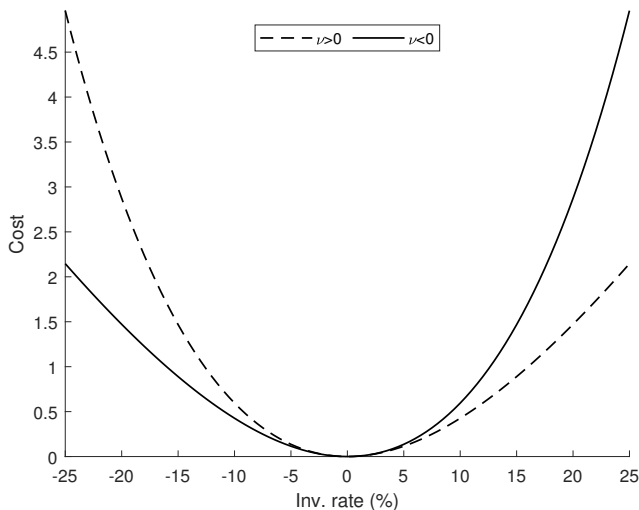
This function is smooth and homogeneous of degree one, hence it satisfies the requirements for the firm value decomposition result in Subsection 3.3. To help its interpretation, Figure 1 plots this function for the one-capital input case. The parameter  $\theta_i$  is similar to the single parameter in the baseline specification and controls the size of the adjustment costs of input  $i$ . The novel parameter here is  $v_i$  which controls the degree of asymmetry of the function. When  $v_i > 0$ , it is more costly to disinvest (partial irreversibility) than it is to invest. When  $v_i < 0$ , it is more costly to invest than it is to disinvest. When  $v_i \rightarrow 0$ , the function converges to our standard quadratic adjustment cost specification.<sup>5</sup> Thus, by estimating the parameter  $v_i$ , we allow the data to uncover the importance of asymmetric adjustment costs for our results. Note that, due to the way in which we calculate investment in the intangible capital inputs, the gross investment rates of these inputs are never negative. Thus, even though the asymmetry parameters for the intangible capital inputs can be

<sup>5</sup>Using l'Hopital's rule,  $\lim_{v \rightarrow 0} \frac{\theta}{v^2} \left[ \exp \left( -v \frac{I}{K} \right) + v \frac{I}{K} - 1 \right] K = \frac{\theta}{2} \left( \frac{I}{K} \right)^2 K$ .



Figure 1: Asymmetric Adjustment Costs Function

This figure shows the asymmetric adjustment costs function specification  $C = \frac{\theta}{v} [\exp(-v\frac{I}{K}) + v\frac{I}{K} - 1] K$ , using a slope adjustment cost parameter  $\theta = 1$ , a capital stock of  $K = 1$ , with curvatures of  $\nu = -5$  (solid) and  $\nu = 5$  (dashed). When  $v > 0$  it is more costly to desinvest than to invest (to capture irreversibility), and vice versa when  $v < 0$ .



estimated, they should be interpreted with caution because the identification of the functional form  
 240 of the adjustment costs of these inputs is only based on the positive side of investment. Hence,  
 in what follows, we focus most of our discussion on the asymmetry parameters  $v$  for the physical  
 capital and labor inputs.

Table 3 reports the parameter estimates and fit of the model with asymmetric adjustment costs.<sup>6</sup>  
 The evidence of asymmetry for physical capital is not strong in our sample. In low-skill industries,  
 245 the asymmetry parameter is positive,  $v_K = 0.21$ , consistent with some irreversibility of investment,  
 but in high-skill industries the parameter is negative,  $v_K = -0.25$ . In both types of industries,  
 however, we cannot reject the hypothesis that this asymmetry parameter is zero, that is, that the  
 physical capital adjustment costs function is symmetric, as in the baseline specification. For labor,

<sup>6</sup>Note that the estimation using this adjustment cost specification can no longer be performed using linear OLS. Here, minimizing the objective function in equation (23) in the main draft requires non linear least squares (NLLS) estimation. We compute bootstrapped standard errors that are robust to cross-sectional and time-series correlation using 20% of the sample with replacement. As shown by Cameron and Miller (2010) bootstrapping controls for the fact that errors can be correlated across portfolios and within portfolios over time.

there is evidence of some degree of irreversibility in high-skill industries with  $\nu_L = 2.16$  (and this value is more than 4.3 standard errors from zero), but not in low-labor-skill industries with  $\nu_L = 1.19$  (but we cannot reject the hypothesis that this parameter is zero). Thus, in high-skill industries, it is more costly to decrease the labor force (i.e., fire workers) than it is to increase it.

Turning to the analysis of the impact on model fit, Table 3 shows that, by using the asymmetric adjustment costs function specification, the time-series  $R^2$  of the model increases by 2 percentage points relative to the baseline quadratic adjustment cost specification, from 38% to 40%. The improvement in high-skill industries is slightly larger. In high-skill industries, using the asymmetric adjustment costs function specification, the time-series  $R^2$  of the model increases by 6 percentage points relative to the baseline quadratic adjustment cost specification, from 60% to 66%. This improved fit comes mostly from the asymmetry in the labor adjustment costs discussed above.

Taken together, allowing for asymmetry in the adjustment costs function seems to have only a small impact on the quality of the model fit in our sample, especially in low-skill industries.

### 3.1.2 Flexible Curvature Adjustment Costs

In the benchmark specification, for parsimonious reasons, we fix the curvature of the adjustment costs function to be equal to two and only estimate the slope adjustment cost parameter. As a robustness check, in this appendix, we consider a more flexible adjustment costs specification that allows for the joint estimation of curvature and slope. Specifically, we consider the following functional form for the adjustment costs function:

$$C_{it} = \frac{\theta_P}{\nu_P} \left| \frac{I_{it}^P}{K_{it}^P} \right|^{\nu_P} K_{it}^P + \frac{\theta_L}{\nu_L} \left| \frac{H_{it}}{L_{it}} \right|^{\nu_L} W_{it} L_{it} + \frac{\theta_K}{\nu_K} \left| \frac{I_{it}^K}{K_{it}^K} \right|^{\nu_K} K_{it}^K + \frac{\theta_B}{\nu_B} \left| \frac{I_{it}^B}{K_{it}^B} \right|^{\nu_B} K_{it}^B, \quad (18)$$

in which  $W_{it}$  is the wage rate (which the firm takes as given),  $\theta_P, \theta_L, \theta_K, \theta_B > 0$  are the slope adjustment costs parameters, and  $\nu_P, \nu_L, \nu_K, \nu_B > 1$  are the curvature adjustment costs parameters. Note that this specification reduces to the quadratic functional form we use in the benchmark model when the curvature parameters are equal to two. The absolute value specification of the adjustment costs function allows for negative investment rates and improves the stability of the estimation of the curvature parameters.<sup>7</sup> This functional form generalizes the one-physical-capital input functional

<sup>7</sup>When the curvature parameters are greater than one,  $\nu_i > 1$ , this function is continuous along its entire domain

Table 3: Parameter Estimates and Model Fit with an Asymmetric Adjustment Cost Specification  
This table reports the parameter estimates and measures of fit for the model with adjustment costs function that allows for asymmetric costs. The estimation uses 40 portfolios sorted based on proxies of the lagged values of the inputs (10 portfolios for each input).  $\theta_P$ ,  $\theta_L$ ,  $\theta_K$  and  $\theta_B$  are, respectively, the physical capital, labor, knowledge capital, and brand capital adjustment cost parameters.  $\nu_P$ ,  $\nu_L$ ,  $\nu_K$  and  $\nu_B$  are, respectively, the physical capital, labor, knowledge capital and brand capital asymmetry adjustment cost parameters. s.e. stands for bootstrapped standard errors.  $XS - R^2$  is the cross-sectional  $R^2$ ,  $TS - R^2$  is the time-series  $R^2$ , and  $m.a.e./\sqrt{VR}$  is the mean absolute valuation error scaled by the absolute value of the ratio. Model-implied input-shares ( $\mu$ ) are computed at the aggregate-level according.  $CX/Y$  is the ratio (in percent) of the implied input adjustment costs-to-sales ratio, computed as the time series average of the cross sectional median of this value. The sample consists of firm-level annual data from 1975 to 2016.

	All Firms	Low Skill	High Skill
	(1)	(2)	(3)
Parameter estimates			
Slope			
$\theta_P$	2.33	4.45	3.02
s.e.	[1.32]	[2.31]	[1.42]
$\theta_L$	15.21	9.32	13.41
s.e.	[1.54]	[2.95]	[1.28]
$\theta_K$	18.19	30.29	16.94
s.e.	[1.87]	[6.58]	[1.70]
$\theta_B$	1.42	29.17	0.45
s.e.	[2.94]	[5.86]	[2.14]
Asymmetry			
$\nu_P$	-0.37	0.21	-0.25
s.e.	[0.28]	[0.79]	[0.28]
$\nu_L$	2.55	1.19	2.16
s.e.	[0.56]	[1.29]	[0.50]
$\nu_K$	1.73	2.31	1.47
s.e.	[0.57]	[1.49]	[0.51]
$\nu_B$	-3.57	9.32	-4.96
s.e.	[2.49]	[2.15]	[2.00]
Model fit			
$XS - R^2$	0.94	0.90	0.94
$TS - R^2$	0.67	0.40	0.66
$m.a.e./\sqrt{VR}$	0.20	0.31	0.20
Firm-value decomposition (in %)			
$\bar{\mu}^P$ : Physical capital	31.87	38.48	31.76
$\bar{\mu}^L$ : Labor	20.97	13.81	19.04
$\bar{\mu}^K$ : Knowledge capital	39.28	22.29	43.93
$\bar{\mu}^B$ : Brand capital	7.88	25.42	5.27
Realized adjustment costs (in %)			
$CP/Y$ : Physical capital	1.46	1.44	2.13
$CL/Y$ : Labor	19 7.58	2.98	7.58
$CK/Y$ : Knowledge capital	12.41	3.21	15.74
$CB/Y$ : Brand capital	0.31	2.56	0.11

form specification used in BXZ to multiple inputs.<sup>8</sup> Finally, note that this functional form also  
 275 assumes symmetry across positive and negative adjustments.<sup>9</sup>

The adjustment costs function in equation (18) implies that the shadow prices of the capital  
 inputs are given by:

$$q_{it}^P \equiv 1 + (1 - \tau_t)\theta_P \left| \frac{I_{it}^P}{K_{it}^P} \right|^{\nu_K - 1} \text{sign} \left( \frac{I_{it}^P}{K_{it}^P} \right) \quad (19)$$

$$q_{it}^L \equiv (1 - \tau_t)\theta_L \left| \frac{H_{it}}{L_{it}} \right|^{\nu_L - 1} \text{sign} \left( \frac{H_{it}}{L_{it}} \right) W_{it} \quad (20)$$

$$q_{it}^K \equiv (1 - \tau_t) \left[ 1 + \theta_K \left| \frac{I_{it}^K}{K_{it}^K} \right|^{\nu_K - 1} \text{sign} \left( \frac{I_{it}^K}{K_{it}^K} \right) \right] \quad (21)$$

$$q_{it}^B \equiv (1 - \tau_t) \left[ 1 + \theta_B \left| \frac{I_{it}^B}{K_{it}^B} \right|^{\nu_B - 1} \text{sign} \left( \frac{I_{it}^B}{K_{it}^B} \right) \right]. \quad (22)$$

We use the sign function to express the equilibrium shadow prices of each of the capital inputs in  
 a compact manner, that is, using one equation, instead of a piece-wise function. This is because,  
 280 given the absolute value specification, the signs associated with the investment and hiring rate terms  
 switch depending on whether the input-specific investment or hiring rate is positive or negative.  
 Using these shadow prices, as in the benchmark model, we can write the firm's valuation ratio as:

$$VR_{it} = q_{it}^P \frac{K_{it+1}^P}{A_{it+1}} + q_{it}^L \frac{L_{it+1}}{A_{it+1}} + q_{it}^K \frac{K_{it+1}^K}{A_{it+1}} + q_{it}^B \frac{K_{it+1}^B}{A_{it+1}}. \quad (23)$$

The left-hand side (LHS) of equation (23) can be directly measured in the data from equity price  
 data and debt data (and measures of the capital stocks). The right hand side (RHS) of equation  
 285 (23) is the predicted valuation ratio from the model,  $\widehat{VR}_{it}$ , which depends on the model parameters.

Table (4) columns (1) to (3) displays the estimated parameters for slope and curvature for the  
 model with three capital inputs and labor in the pooled sample (all firms), and for low- and high-  
 skill industries separately. Looking at the estimated curvature parameters, one can note that the

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including at zero since the left and right derivatives at zero coincide. See also Kogan and Papanikolaou (2012) for a  
 similar specification in the context of a one-physical-capital input model.

<sup>8</sup>Although not explicitly stated in their paper, BXZ also use an absolute value specification to deal with negative  
 investment rates observed in the data.

<sup>9</sup>Note that the estimation can no longer be performed using OLS. Therefore we use non-linear least squares (NLLS)  
 and then compute bootstrapped standard errors that are robust to cross-sectional and time-series correlation using  
 20% of the sample with replacement. As shown by Cameron and Miller (2010), bootstrapping controls for the fact  
 that errors can be correlated across portfolios and within portfolios over time.

values are around two, empirically justifying the simplification used in the benchmark model. Note  
290 also that, comparing the fit of this model with the quadratic version in Table 3 columns (1) to (3)  
in the main draft, we can conclude that estimating the curvature does not significantly improve the  
model fit. Table 4 columns (4) to (6) displays the estimated parameters for slope and curvature for  
the model with only physical capital in the pooled sample (all firms), and for the low- and high-skill  
industries separately. Comparing with the quadratic cost counterpart in Table 3 columns (4) to (5)  
295 we see that allowing for curvature estimation also only marginally improves the fit of the model.

The last two panels in Table (4) report the input-share decomposition and adjustment costs  
using the specification in equation (18) and the previous parameter estimates. Overall, the message  
is quite similar to the one in the main draft, namely, that physical capital accounts for a relatively  
small share of firm value, and that labor and intangible capital are important to properly decompose  
300 the market value of the firm.

### 3.2 Alternative Test Assets, Estimation Method, and Samples

In the baseline estimation, we use 40 portfolios (10 portfolios for each of the 4 portfolio sorts) as  
test assets. In this section we show how the selection of test assets (portfolios), estimation method  
and sample affect the model fit and conclusions. For parsimonious reasons, we focus here on the  
305 estimation results across all firms. In Section 3.2.1 we estimate the model using a larger number  
of portfolios than in the baseline estimation. In Section 3.2.2, we consider an alternative industry  
classification and a different sorting variable for the portfolios. In Section 3.2.3 we estimate the  
model directly using firm-level data (as opposed to performing the estimation using portfolios)  
and targeting the valuation ratio moment, as in the baseline estimation. Section 3.2.4 repeats the  
310 previous estimation exercise also using firm-level data but now using the investment Euler equations  
as the target moments instead of the valuation ratio (and hence, it does not use any asset price  
data). Finally, in Section 3.2.5 we re-estimate a restricted version of the model without knowledge  
capital using the sub-sample of firms that were excluded from the main sample due to missing (or  
always zero) R&D expenses data.

Table 4: Parameter Estimates with Flexible Curvature

This table reports estimation results and measures of fit for the model with the curvature estimation for the benchmark model and also the one-physical capital input model. The estimation uses forty portfolios based on the lagged investment/hiring (ten of each type of input).  $\theta_P$ ,  $\theta_L$ ,  $\theta_K$  and  $\theta_B$  are respectively, the physical capital, labor, knowledge capital and brand capital slope adjustment costs parameters.  $\nu_P$ ,  $\nu_L$ ,  $\nu_K$  and  $\nu_B$  are, respectively, the physical capital, labor, knowledge capital and brand capital curvature adjustment costs parameters. s.e. stands for bootstrapped standard errors. m.a.e./|VR| is the mean absolute valuation error scaled by the absolute value of the ratio. The value that is attributed to each input ( $\mu$ ) is presented in the third panel. The second panel report the costs, where  $CX/Y$  is the ratio (in percent) of the implied input adjustment costs-to-sales ratio. The sample is annual data from 1975 to 2016.

	Baseline			$K^P$		
	All	Low	High	All	Low	High
	Firms	Skill	Skill	Firms	Skill	Skill
	(1)	(2)	(3)	(4)	(5)	(6)
	Parameter Estimates					
Slope						
$\theta_P$	2.88	3.40	3.11	28.37	17.35	29.13
s.e.	[1.11]	[1.33]	[1.12]	[0.88]	[1.62]	[0.87]
$\theta_L$	10.16	4.89	10.73			
s.e.	[1.04]	[1.34]	[1.08]			
$\theta_K$	9.12	12.44	9.26			
s.e.	[1.32]	[5.43]	[1.35]			
$\theta_B$	4.23	4.48	4.17			
s.e.	[3.66]	[4.13]	[4.14]			
Curvature						
$\nu_P$	2.45	2.21	2.41	1.95	1.64	1.94
s.e.	[0.16]	[0.42]	[0.17]	[0.04]	[0.08]	[0.05]
$\nu_L$	1.97	1.42	2.09			
s.e.	[0.13]	[0.21]	[0.14]			
$\nu_K$	1.68	1.65	1.7			
s.e.	[0.10]	[0.28]	[0.10]			
$\nu_B$	2.34	1.49	2.56			
s.e.	[0.43]	[0.48]	[0.44]			
Model Fit						
XS- $R^2$	0.93	0.90	0.93	0.73	0.58	0.74
TS- $R^2$	0.62	0.42	0.61	0.23	0.12	0.20
m.a.e./VR	0.22	0.31	0.22	0.32	0.37	0.33
Firm value decomposition - Aggregate (in %)						
$\bar{\mu}^P$ : Physical capital	31.06	34.71	30.01	100.00	100.00	100.00
$\bar{\mu}^L$ : Labor	20.95	22.08	18.43			
$\bar{\mu}^K$ : Knowledge capital	39.93	20.96	45.5			
$\bar{\mu}^B$ : Brand capital	8.06	22.24	6.06			
Realized adjustment costs (in % of annual sales)						
$CP/Y$ : Physical capital	0.79	0.68	1.09	18.91	13.12	22.22
$CL/Y$ : Labor	6.24	6.48	5.69			
$CK/Y$ : Knowledge capital	13.2	3.21	16.89			
$CB/Y$ : Brand capital	0.34	2.09	0.23			

### 315 3.2.1 Changing the Number of Portfolios

Here, we consider 80 portfolios (20 portfolios for each portfolio sort) as test assets, and investigate the impact of changing the number of portfolios on the results. Table 5, column (2) reports the estimation results using this larger number of portfolios as test assets. The point estimates appear to be very similar in magnitude to the point estimates in the baseline estimation. As a result, the  
320 model fit and model-implied firm-value decomposition are all quite similar to those obtained in the baseline estimation of the model. This analysis suggests that the point estimates in the baseline estimation are robust to a reasonable variation of the number of portfolios used in the estimation.

Table 5: Alternative Test Assets, Estimation Method, and Samples

This table reports the parameter estimates, measures of fit, and model-implied input-shares ( $\mu$ ) across alternative empirical procedures.  $\theta_P$ ,  $\theta_L$ ,  $\theta_K$ , and  $\theta_B$  are, respectively, the physical capital, labor, knowledge capital, and brand capital slope adjustment cost parameters. s.e. stands for Newey-West standard errors with three lags.  $XS - R^2$  is the cross-sectional  $R^2$ ,  $TS - R^2$  is the time-series  $R^2$ , and  $m.a.e./VR$  is the mean absolute valuation error scaled by the absolute value of the ratio. Column (1) reports the baseline estimation results reported in the main draft using 40 portfolios (Baseline). Column (2) reports the estimation results using 80 portfolios sorted based on proxies of the lagged values of the inputs (80 portfolios). Column (3) reports the estimation results using 17 Fama-French industries as the portfolios (17 Ind.). Column (4) reports the estimation results by performing the estimation at the firm- (not portfolio-) level (Firm-Level) and using the valuation ratio as the target moment. Column (5) reports the estimation results by performing the estimation also at the firm-level (Firm-Level) but using the investment Euler Equations as the target moments. Columns (6) reports the benchmark estimation in the sample of firms with missing (or always zero) R&D expense data, using a model with physical capital, labor, and brand capital as the inputs (Non R&D Firms). The results are reported for the sample of all firms. The sample consists of firm-level annual data from 1975 to 2016.

	Baseline		80 Portfolios		17 Industries		Firm-Level		Firm-Level		Non-R&D Firms	
	Valuation	Moments	Valuation	Moments	Valuation	Moments	Valuation	Euler	Valuation	Equations	Valuation	Moments
	(1)		(2)		(3)		(4)		(5)		(6)	
$\theta_P$	1.50		2.22		2.66		3.72		3.19		5.65	
s.e.	[1.00]		[0.73]		[0.75]		[0.37]		[0.20]		[0.99]	
$\theta_L$	11.26		9.95		6.98		5.30		6.61		7.39	
s.e.	[0.69]		[0.55]		[1.03]		[0.27]		[0.61]		[0.48]	
$\theta_K$	12.47		12.92		15.16		7.60		9.31			
s.e.	[0.77]		[0.61]		[1.85]		[1.57]		[0.81]			
$\theta_B$	3.24		4.40		11.09		13.22		13.23		5.97	
s.e.	[2.05]		[1.47]		[3.33]		[1.18]		[1.35]		[2.47]	
	Parameter estimates											
	Model fit											
$XS - R^2$	0.94		0.94		0.65		0.26		-		0.94	
$TS - R^2$	0.61		0.58		0.34		0.15		-		0.72	
$m.a.e./VR$	0.22		0.24		0.30		0.64		-		0.25	
	Firm-value decomposition (in %)											
$\bar{\mu}^P$ : Physical	30.36		31.88		30.98		38.77		30.15		56.65	
$\bar{\mu}^L$ : Labor	22.53		19.55		12.39		11.45		12.20		30.48	
$\bar{\mu}^K$ : Knowledge	38.28		38.56		39.75		27.79		34.03			
$\bar{\mu}^B$ : Brand	8.83		10.02		16.87		21.98		23.62		12.87	



Table 6: Estimated Adjustment Costs for Alternative Test Assets, Estimation Method, and Samples

This table complements the results from the previous table, and reports the realized adjustment costs implied by the alternative model specifications, estimation procedure, and sample.

	Baseline	80 Portfolios	17 Industries	Firm-Level	Firm-Level	Non R&D
	Valuation Moments (1)	Valuation Moments (2)	Valuation Moments (3)	Valuation Moments (4)	Euler Equations (5)	Valuation Moments (6)
<i>CP/Y</i> : Physical capital	0.90	1.34	1.78	2.49	1.52	2.40
<i>CL/Y</i> : Labor	6.46	5.71	4.39	3.33	3.19	6.90
<i>CK/Y</i> : Knowledge capital	10.05	10.41	13.18	6.60	6.77	
<i>CB/Y</i> : Brand capital	0.49	0.66	1.80	2.15	2.25	1.27
	Realized adjustment costs (in %)					

### 3.2.2 Alternative Portfolio Sorts and Industry Classification

In the baseline analysis we estimate the model using portfolios sorted on proxies for the firms' lagged values of each input. In addition, we split the sample into low- and high-skill industries according to the average share of high-skilled workers in each industry. Naturally, the model can be estimated using other portfolio sorts, and also using other industry classifications.

To check the robustness of our main findings to both the portfolio sorting variable and the industry classification, here we report the estimation results using two alternative procedures. In the first procedure, we estimate the model parameters using 15-industry portfolios following the 17-industry Fama and French industry classification (we exclude two industries due to data availability), instead of sorting the portfolios on proxies for the firms' lagged values of each input.<sup>10</sup> The results from this analysis allow us to check the robustness of the findings to the portfolio sorting variable(s). Implicit in this analysis is the assumption that the adjustment costs technology is similar across these industries (we estimate only one set of parameters for all firms). Thus, we also consider a second alternative procedure in which we estimate the model parameters using the same sorting variables of the baseline estimation but perform the estimation separately within each Fama and French industry. The results from this analysis allow us to check the robustness of the findings to the industry classification. To save space, given the large set of results obtained using this second procedure, we discuss here a brief summary of the main results and report the complete analysis using this procedure in Subsection 4.2 in this appendix. Further, we report only the input-shares computed using the aggregate input-share measure.

Table 5, column (3) reports the estimation results using the 15-industry portfolios. The point estimates are similar to those obtained in the baseline estimation. The only noticeable differences

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<sup>10</sup>We use the 17-industry classification posted on Kenneth French's website. We exclude the industries 14–Utilities and 16–Financial firms due to data availability and sample restrictions. We are left with the following fifteen industries: 1–Food, 2–Mines (Mining and Minerals), 3–Oil (Oil and Petroleum Products), 4–Clths (Textiles, Apparel & Footware), 5–Durbl (Consumer Durables), 6–Chems (Chemicals), 7–Cnsum (Drugs, Soap, Perfumes, Tobacco), 8–Cnstr (Construction and Construction Materials), 9–Steel (Steel Works, etc.), 10–FabPr (Fabricated Products), 11–Machn (Machinery and Business Equipment), 12–Cars (Automobiles), 13–Trans (Transportation), 15–Rtail (Retail Stores), 17–Other.

are the slope coefficient on brand capital that is larger than in the baseline case ( $\theta_B = 11.09$  here versus  $\theta_B = 3.24$  in the baseline estimation), and the slope coefficient on labor that is smaller than  
350 in the baseline case ( $\theta_L = 6.98$  here versus  $\theta_L = 11.26$  in the baseline estimation). As a result, the estimated share of brand capital for firm value is slightly higher here than in the baseline model ( $\mu_B = 16.87$  here versus  $\mu_B = 8.83$  in the baseline estimation), while the estimated share of labor capital for firm value is slightly lower here than in the baseline model ( $\mu_L = 12.39$  here versus  $\mu_L = 8.83$  in the baseline estimation). More important, the results confirm the importance of  
355 the non-physical capital inputs for firm value. Similar to the baseline estimation, the non-physical capital inputs account for roughly 70% of the firm’s market value.

The estimation of the model for the different Fama and French industries provides further support for the importance of the non-physical capital inputs for firm value. In Subsection 4.2 in this appendix we show that although the estimates of the adjustment costs parameters vary across  
360 industries, the importance of the non-physical capital inputs persists. The average share of the non-physical inputs ranges from a minimum of 19% in the industry classified as “other”, to a maximum of 72% in the high-tech industry. In addition, the analysis of the input-shares in each industry and over time, confirms that the decline in the share of physical capital and the corresponding increase in the share of knowledge capital, also observed in the baseline estimation, also persists across the  
365 Fama and French industries. Thus, the decline in the physical-capital share and the increase in the knowledge capital share is not driven by changes in the industry composition in the U.S. economy, but rather seems to be a trend in the overall economy.

### 3.2.3 Firm-level Estimation

We perform the baseline estimation using portfolio-level moments. Alternatively, we can estimate  
370 equation (24) by ordinary least squares directly on firm-level data. The advantage of this latter approach is that it does not require us to take a stand regarding a particular sorting variable to create the portfolios. The disadvantage is that this approach is more sensitive to noise in the firm-level data.

Table 5, column (4), reports the estimation results using firm-level data. As expected, the  
375 parameter estimates differ somewhat from the baseline estimation. The main noticeable difference

is the smaller estimate of the labor adjustment cost parameter, and the larger estimate of the brand capital adjustment cost parameter. This suggests that the noise in the labor input may be more severe than the noise in the other inputs. As a result, the estimated share of labor for firm value is smaller here than in the baseline model, and the estimated share of brand capital for firm value is larger here than in the baseline model.<sup>11</sup>

More important, the estimation results using directly the firm-level data confirm the importance of non-physical capital for firm value. Similar to the baseline estimation, the non-physical capital inputs account for a substantial fraction, approximately 62% of firm value.

### 3.2.4 Alternative Estimation Method: Firm-Level Euler Equation Approach

Here we check the robustness of our main findings to the estimation method, in particular, estimating the model parameters using the investment Euler equations as the target moments, instead of the valuation ratios as in the baseline approach.

Rearranging the first order conditions with respect to investment and hiring leads to the following four Investment-Euler equations (using the same notation as in the main draft):

$$E_{KP} : 1 + (1 - \tau_t)\theta_P \left( \frac{I_{it}^P}{K_{it}^P} \right) = E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left( \alpha_P \frac{Y_{it+1}}{K_{it+1}^P} + \frac{\theta_P}{2} \left( \frac{I_{it+1}^P}{K_{it+1}^P} \right)^2 \right) + \delta_{it+1}^P \tau_{t+1} + (1 - \delta_{it+1}^P)(1 + (1 - \tau_{t+1})\theta_P \left( \frac{I_{it+1}^P}{K_{it+1}^P} \right)) \right] \right]$$

$$E_L : (1 - \tau_t)\theta_L \left( \frac{H_{it}}{L_{it}} \right) W_{it} = E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left( \alpha_L \frac{Y_{it+1}}{L_{it+1}} + \frac{\theta_L}{2} \left( \frac{H_{it+1}}{L_{it+1}} \right)^2 W_{it+1} - W_{it+1} \right) + (1 - \delta_{it+1}^L)(1 - \tau_{t+1})\theta_L \left( \frac{H_{it+1}}{L_{it+1}} \right) W_{it+1} \right] \right]$$

$$E_{KK} : (1 - \tau_t) \left[ 1 + \theta_K \left( \frac{I_{it}^K}{K_{it}^K} \right) \right] = E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left( \alpha_K \frac{Y_{it+1}}{K_{it+1}^K} + \frac{\theta_K}{2} \left( \frac{I_{it+1}^K}{K_{it+1}^K} \right)^2 \right) + (1 - \delta_{it+1}^K)(1 - \tau_{t+1}) \left[ 1 + \theta_K \left( \frac{I_{it+1}^K}{K_{it+1}^K} \right) \right] \right] \right]$$

<sup>11</sup>As implied by the model, we restrict the intercept to be zero in the regression analysis. This restriction also prevents artificial improvement of the model fit. In unreported results (available upon request), we find that including an intercept to the firm-level regression does not significantly improve the model fit.

$$E_{KB} : (1-\tau_t) \left[ 1 + \theta_B \left( \frac{I_{it}^B}{K_{it}^B} \right) \right] = E_t \left[ M_{t+1} \left[ (1-\tau_{t+1}) \left( \alpha_B \frac{Y_{it+1}}{K_{it+1}^B} + \frac{\theta_B}{2} \left( \frac{I_{it+1}^B}{K_{it+1}^B} \right)^2 \right) + (1-\delta_{it+1}^B)(1-\tau_{t+1}) \left[ 1 + \theta_B \left( \frac{I_{it+1}^B}{K_{it+1}^B} \right) \right] \right] \right]$$

To make the labor Euler equation  $E_L$  stationary, we divide both sides of the equation by current wages  $W_{it}$ . To facilitate the estimation of the key adjustment cost parameters, we substitute in these equations the operating profit function parameters (the  $\alpha_i$ 's) estimated using the Akerberg, Caves, and Frazer (2015) methodology based on value added (the operating profit function estimation method is discussed in Section 2.2 in this appendix). In addition, we assume a simple discount factor equal to  $M = 1/(1+r)$  and  $r = 5\%$ . We then estimate these four Euler equations using a standard approach (e.g. Whited (1992), among many others). Specifically, we replace the expectation operator with a white noise expectational error, which is uncorrelated with any information known at time  $t$ . We then estimate each set of Euler equations separately by the generalized method of moments (GMM), using a constant as the only instrument (that is, the identification assumption is that the expectation error is on average zero).

Unlike our baseline portfolio-level estimation approach, firm-level estimation is more sensitive to undetected outliers in noisy firm-level data. To mitigate such concerns we clean further the data for the firm-level Euler equation estimation. In particular, we only include firms that have: non-negative sales, a depreciation rate of physical capital less than 100%, a minimum of 5 observations, a minimum of 50 workers, and minimum capital stocks (physical, knowledge, and brand) of \$100,000. We also eliminate observations with extreme values of the marginal products of the intangible capital inputs.<sup>12</sup> These data requirements leaves us with 2,088 firms for the estimation.

The adjustment cost parameter estimates, and implied firm value shares and input adjustment costs, are presented in Table 5 and Table 6 (column 5). We can see that the point estimates of the adjustment cost parameters are similar to the parameter estimates obtained when we target the valuation ratio moments also using the firm-level data (column 4) and also broadly in line with the other robustness checks reported. As noted above, the firm-level data is subject to substantial

<sup>12</sup>Specifically, we eliminate observations in which the marginal product of knowledge capital or brand capital is more than 20 times the marginal product of physical capital. We note that these variables (including depreciation rate and sales data) are not directly used in our estimation method, hence these criteria do not affect our previous results.

noise, so the firm-level approach is likely to be more affected by noise and undetected outliers. Nevertheless, although the point estimates of the adjustment cost parameters are not identical to the baseline portfolio estimates, the implied value decomposition, the central result in our draft, and also the implied magnitude of the input adjustment costs, do not change much relative to the  
425 baseline estimation. In particular, the contribution of the non-physical capital inputs for firm value is quite substantial, roughly 70% (which is similar to the value reported in the main draft).<sup>13</sup>

While the Euler equation approach is a valid alternative estimation method, in the context of our application the estimation approach in the main draft has the advantage of being parsimonious, not requiring the explicit estimation of cash flows (e.g., the factor share parameters in the operating  
430 profit function), nor taking a stand on the stochastic discount factor.<sup>14</sup> In addition, our approach uses asset price data which we is important in our application because the goal is to understand firm valuation in financial markets. Finally, our estimation approach using portfolio-level data is less sensitive to noise in the data.

### 435 3.2.5 Alternative Samples

As discussed in Section 4.4 in the main text, in the main sample, we drop firms that never report (or always report zero) R&D expenses. Ignoring these firms may not be efficient for the purposes of our analysis, however, because these firms may be informative about the importance of the non-physical capital inputs (labor and brand capital) for firm value. Thus, here we estimate a (restricted) version  
440 of the model with physical capital, labor, and brand capital only, using the sample of firms that were excluded from the main sample due to missing (or always zero) R&D expenses data. This alternative sample includes 6,541 firms, and 60,316 firm-year observations.

Table 5, columns (6), reports the estimation results obtained using this alternative sample of non-R&D firms. The model fit is even better than the baseline sample/model. The times-series  $R^2$

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<sup>13</sup>The similarity between the point estimates reported in columns 4 (firm-level targeting the valuation ratio moments) and column 5 (firm-level targeting the investment Euler equations) suggests that endogeneity concerns in the investment-q relationship do not seem to be a major issue in our analysis.

<sup>14</sup>See, for example, Bond and Van Reenen (2007) (Section 3) for an interesting analysis of the advantages and disadvantages of different investment demand estimation approaches. Also, as discussed in the Related Literature section 2 of the main draft, our approach of using asset price data (valuation ratios) to estimate model parameters is closely related to previous work in the area (in particular, see Belo, Xue, and Zhang (2013) and Merz and Yashiv (2007)).

445 is 72%, whereas in the baseline sample it is 61%.

The share of labor ranges is 30.5% , whereas the share of brand capital is 12.9%. The share of physical capital for non-R&D firms is significantly higher than in the baseline model: 56.7% here versus 30.7% in the baseline sample. This higher share relative to the baseline sample is perhaps not surprising given that, by definition, the non-R&D firms have zero knowledge capital, which (across  
450 most specifications) is the non-physical capital input that contributes the most for firm value in the baseline sample. In addition, the firms that do not perform R&D are likely to be firms from the “old economy,” and naturally rely less on innovation and other intangibles, and more on installed physical capital.

Taken together, the average contribution of the non-physical capital inputs for firm value in this  
455 alternative sample is still more than 42% of firms’ market value. Although this share is smaller than in the baseline model, it is still substantial, thus providing additional support for the importance of the non-physical capital inputs for firm value.

### 3.3 An Alternative Intangible Capital Stock Based on SG&A Data (Organization Capital)

460 In the main draft, we measure intangible capital using expenditure data on research and development and on advertising. Therefore, to be included in our analysis firms must report these two types of expenditures. Here, we consider an alternative measure of intangible capital (organization capital) that does not differentiate between knowledge and brand capital. Specifically, we construct a measure of organization capital based on Selling, General and Administrative (SG&A) expense  
465 data, following Eisfeldt and Papanikolaou (2013). Since this item is more regularly reported we can perform the analysis on a larger sample of firms. Our sample with physical capital, labor and organization capital has 6,974 firms and 77,263 observations.

We construct the firms’ stock of organization capital from past expenditures data on SG&A (Compustat data item XSGA) and using the perpetual inventory method as follows:

$$K_{t+1}^O = K_t^O(1 - \delta^O) \frac{P_{t+1}^O}{P_t^O} + I_{t+1}^O, \quad (24)$$

470 where  $P_t^O$  is the BEA price index for consumption expenditures.<sup>15</sup>

We set organization capital investment to be equal to 30% of SG&A expenditures following Peters and Taylor (2017). To implement the law of motion in equation (24) we must choose an initial stock and a depreciation rate. Using the perpetual inventory method, we set the initial stock to:

$$K_0^O = \frac{I_0^O}{g^O + \delta^O - \pi^O(1 - \delta^O)},$$

475 in which  $I_0^O$  is the firm's investment in organization capital in the first year in the sample, and  $\pi^O$  is the average (net) growth rate of the price index for SG&A, which is 3.3% in the sample period used for the estimation. We let  $g^O$  be industry-specific and set it equal to the average growth rate of the SG&A investments in that industry; in practice, we consider 10 industry-groups based on the level of the labor skill level in that industry. As for the organization capital depreciation rate, we use 20%.  
480 Once we have the initial capital stock, we iterate forward using the appropriate depreciation rate, SG&A expenses, and investment price index. The investment rate on organization capital is then given by the ratio of the current period investment and the beginning of the period corresponding knowledge capital stock  $I_t^O/K_t^O$ .

We estimate the model using a quadratic adjustment costs specification using a sample of ten  
485 portfolios based on each investment/hiring input (total of thirty portfolios). Table 7 first panel displays the estimated slopes. Column (1), reports the point estimates of the adjustment costs parameters in the pooled sample. The estimates of the adjustment costs parameters are  $\theta_P = 1.23$  for physical capital,  $\theta_L = 6.16$  for labor, and  $\theta_O = 9.49$  for organization capital. The second panel displays the model fit. According to the three metrics considered here, the model performs well –  
490 both in the time-series and cross-section dimensions – when estimated across all firms. Columns (2) and (3) display the estimated slopes and fit for low- and high skill industries. All the adjustment costs parameters are positive and we can reject the hypothesis that these parameters are zero. The estimate of the slope adjustment costs parameter for labor and organization capital increase with the average labor-skill of the industry, from  $\theta_L = 2.60$  and  $\theta_O = 5.49$  in low-skill industries  
495 to  $\theta_L = 6.79$  and  $\theta_O = 10.60$  in high-skill industries. Going in the opposite direction, the slope

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<sup>15</sup>Specifically, we use the annual series “Personal Consumption Expenditures: Chain-type Price Index, Index 2009=100” (DPCERG3A086NBEA) provided by the BEA.



adjustment cost parameters for physical capital with the average labor-skill level of the industry. The model fit is particularly good is capturing the time-series variation in the valuation ratios in high-skill industries, with a time series  $R^2$  of 63%, whereas the time series fit in the low skill industry is more modest,  $R^2$  of 44%.

500 The last two panels in 7 use the estimated parameters to calculate the input-share decomposition and adjustment costs. While using SG&A to measure intangible capital does not allow us to differentiate across the two types of intangible (brand and knowledge), overall the decomposition across physical capital, labor and intangible capital stays similar. Using the aggregate input-share measure, in the pooled sample, physical capital accounts for about 32% of firm value while labor  
505 accounts for 16.83% and organization capital for 51.05%. For firms in low-skill industries, labor accounts for about 10% of firms' market value while in high skill industries this number rises to 15.40%. While organization capital accounts for a larger share of firms' value in high skill industries (52% versus 43%), physical capital accounts for more value in low-skill industries (46.88% versus 32.65%). The last panel show the (average) median adjustment costs in each input. Adjusting labor  
510 is more expensive in high-skill (4.27% of annual sales ) than in low-skill industries (1.84% of annual sales). Organization capital is the most expensive input to adjust, accounting for almost 11% of annual sales in the pooled sample.

Table 7: Model Estimation Using Organization Capital (SG&A)

This table reports estimation results and measures of fit for the sample of firms that reports SG&A. The estimation uses thirty portfolios based on the lagged investment/hiring (ten based on each type of input). The estimation is done using the cross-sectional average aggregation method and LS methodology. The first panel reports the estimation results.  $\theta_P$ ,  $\theta_L$ , and  $\theta_O$  are respectively, the physical capital, labor, organization capital slope adjustment costs parameters. s.e. stands for Newey-West standard errors with three lags. The second panel reports measures of fit, m.a.e./|VR| is the mean absolute valuation error scaled by the absolute value of the ratio. The sample is annual data from 1975 to 2016. The last two panels report input-shares and adjustment costs for this sample using the parameter estimates.

	Baseline		
	All Firms	Low Skill	High Skill
	(1)	(2)	(3)
Parameter Estimates			
Slope			
$\theta_P$	1.23	3.80	1.79
s.e.	[1.09]	[0.98]	[0.88]
$\theta_L$	6.16	2.60	6.79
s.e.	[0.86]	[0.75]	[0.81]
$\theta_O$	9.49	5.49	10.60
s.e.	[0.83]	[0.62]	[0.84]
Model Fit			
XS- $R^2$	0.88	0.84	0.90
TS- $R^2$	0.62	0.44	0.63
m.a.e./VR	0.24	0.27	0.24
Firm value decomposition Aggregate (in %)			
$\bar{\mu}^P$ : Physical capital	32.12	46.88	32.65
$\bar{\mu}^L$ : Labor	16.83	9.99	15.40
$\bar{\mu}^O$ : Org. capital	51.05	43.13	51.95
Realized adjustment costs (in % of annual sales)			
$CP/Y$ : Physical capital	0.60	1.20	1.14
$CL/Y$ : Labor	4.04	1.84	4.27
$CO/Y$ : Org. capital	10.99	4.88	14.5

### 3.4 Alternative Intangible Capital Depreciation Rate Specifications

Differently from tangible capital which depreciates due to physical decay or wear and tear, R&D  
515 depreciates because its contribution to a firm’s profit declines over time. The main driving forces  
for such depreciation are obsolescence and competition (Hall (2007)).

While for physical capital – due to tax benefits – firms report the (accounting) depreciation (and  
hence we can compute the depreciation rate), for intangible capital inputs we need to estimate it.  
Depreciation rates are important for our analysis because we do not observe stocks of intangible  
520 capital and hence we use the perpetual inventory method together with expenditure data to calculate  
stocks of knowledge and brand capital.

For the knowledge capital depreciation rate, we use the industry-level rates of R&D assets based  
on the BEA-NSF data estimated by Li (2012) and reported in Table 4, column 3 for each industry.  
Li’s paper develops a forward looking profit model with gestational lags to derive the depreciation  
525 rates. This strategy levers out Compustat, BEA and NSF data to estimate – for the first time – a  
complete set of R&D depreciation rates for major U.S. high-tech industries. Table 2 of Li’s paper  
shows that her estimates are largely in line with many single industry studies. For those industries  
not reported by Li (2012), we follow Peters and Taylor (2017) and use 15%. For brand capital we  
follow Vitorino (2014) and set depreciation rate at 20%.

530 Since the depreciation rates affect the capitalization of the stocks, and hence the estimated  
adjustment costs and value shares, here we perform robustness tests regarding the depreciation  
rates. We redo our analysis for three different levels of depreciation of knowledge capital – (0.5, 1  
and 1.5) times the value used in the calculations of the results in the paper – and brand capital (10%,  
20%, 30%). This leads to 9 possible combinations of the knowledge and brand capital depreciation  
535 rates. The number reported in the center of each table, in bold, is equivalent to the combination  
used in the specification in the main draft.

Table 8 shows the median of the knowledge and brand investment rates and input stocks.  
Because changes in the depreciation rates of intangible capital change the scale of the inputs, we  
also report the scaled physical capital and labor stocks. Table 9 shows the parameter estimates  
540 and Table 10 the model fit. An decrease in depreciation rate leads to lower investment rate of that  
particular input (because we consider gross investment) and higher stock of the input, as one can

observe from the descriptive statistics in Table 8.

Although the estimated parameters are very similar, comparing across rows or columns we observe that – especially for knowledge and brand capital parameters – this lower (higher) investment rate leads to lower (higher) adjustment cost parameters. The lower parameters and investment rates as a result of the lower depreciation rate pulls adjustment cost and shares down. But this decrease in the depreciation rates leads to larger stocks of the input thus pulling shares and adjustment costs up. The opposite direction of these components generates stability in the shares reported in Table 11. Finally, Table 12 displays the adjustment costs estimated as a share of sales, as the size of the adjustment varies with depreciation rate (lower depreciation, smaller investment), the costs for knowledge and brand mechanically ends up having larger variation. To allow for a proper comparison, we calculate in Table 13 the adjustment costs, evaluated at the same investment rate of 10%. The table shows that the adjustment costs are stable.

Overall, the results reported here show that the value decomposition and adjustment costs estimates that we report in the main draft are robust to reasonable perturbations of the depreciation rates used for the intangible capitals.

Table 8: Descriptive Statistics

This table reports the time-series average of the cross-sectional median, of selected characteristics of the firm level data across all firms in the economy for the different assumptions about the intangible capital depreciation rates. Across columns we vary the depreciation rate of brand capital from 10% to 30%, and across rows we vary the depreciation rate of knowledge capital, from 7% to 23%. The center measure in bold is equivalent to the specification used in the main draft.

	0.10	0.20	0.30
	$I_{it}^K / K_{it}^K$		
0.07	0.19	0.19	0.19
0.15	0.28	<b>0.28</b>	0.28
0.23	0.37	0.37	0.37
	$I_{it}^B / K_{it}^B$		
0.07	0.16	0.25	0.35
0.15	0.16	<b>0.25</b>	0.35
0.23	0.16	0.25	0.35
	$K_{it}^P / A_{it}$		
0.07	0.32	0.35	0.37
0.15	0.38	<b>0.42</b>	0.45
0.23	0.42	0.47	0.50
	$(W_{it-1}L_{it}) / A_{it}$		
0.07	0.48	0.51	0.53
0.15	0.57	<b>0.61</b>	0.64
0.23	0.64	0.69	0.73
	$K_{it}^P / A_{it}$		
0.07	0.44	0.48	0.50
0.15	0.35	<b>0.38</b>	0.40
0.23	0.28	0.31	0.33
	$K_{it}^P / A_{it}$		
0.07	0.13	0.09	0.06
0.15	0.15	<b>0.10</b>	0.08
0.23	0.17	0.12	0.09

Table 9: Parameter Estimates

This table reports the parameter estimates for all firms, low and high skill using the baseline model specification for the different assumptions on depreciation rates. Across columns we vary the depreciation rate of brand capital from 10% to 30%, and across rows we vary the depreciation rate of knowledge capital, from 7% to 23%.  $\theta_P$ ,  $\theta_L$ ,  $\theta_K$  and  $\theta_B$  are, respectively, the physical capital, labor, knowledge capital, and brand capital adjustment cost parameters. The center estimate in bold is equivalent to the specification in the main draft.

	All Firms			Low Skill			High Skill			
	0.10	0.20	0.30	0.10	0.20	0.30	0.10	0.20	0.30	
	$\theta^P$			$\theta^P$			$\theta^P$			
0.07	1.95	2.00	1.80	0.07	0.07	0.07	0.07	2.92	2.97	2.75
0.15	1.71	<b>1.50</b>	1.52	0.15	0.15	0.15	0.15	2.25	<b>2.18</b>	2.27
0.23	1.47	1.11	1.12	0.23	0.23	0.23	0.23	1.84	1.89	1.99
	$\theta^L$			$\theta^L$			$\theta^L$			
0.07	10.67	10.61	10.74	0.07	0.07	0.07	0.07	9.82	9.89	9.97
0.15	11.11	<b>11.26</b>	11.24	0.15	0.15	0.15	0.15	10.53	<b>10.64</b>	10.54
0.23	11.27	11.47	11.44	0.23	0.23	0.23	0.23	10.87	10.8	10.7
	$\theta^K$			$\theta^K$			$\theta^K$			
0.07	11.98	11.81	11.78	0.07	0.07	0.07	0.07	11.65	11.48	11.47
0.15	12.62	<b>12.47</b>	12.43	0.15	0.15	0.15	0.15	12.41	<b>12.28</b>	12.25
0.23	13.06	13.00	12.97	0.23	0.23	0.23	0.23	12.88	12.80	12.79
	$\theta^B$			$\theta^B$			$\theta^B$			
0.07	0.90	3.22	4.44	0.07	0.07	0.07	0.07	0.45	2.05	3.45
0.15	0.82	<b>3.24</b>	4.43	0.15	0.15	0.15	0.15	0.25	<b>2.05</b>	3.32
0.23	1.27	3.73	4.90	0.23	0.23	0.23	0.23	0.42	2.44	3.46

Table 10: R2 across Depreciation Rates

This table reports the measure of fit estimates ( $R^2$ ) for all firms, low and high skill using the baseline model specification for the different assumptions on depreciation rates. Across columns we vary the depreciation rate of brand capital from 10% to 30%, and across rows we vary the depreciation rate of knowledge capital, from 7% to 23%. The center estimate in bold is equivalent to the specification in the main draft.

	All Firms				Low Skill				High Skill		
	0.10	0.20	0.30		0.10	0.20	0.30		0.10	0.20	0.30
0.07	0.94	0.94	0.94	0.07	0.94	0.94	0.94	0.07	0.93	0.94	0.94
0.15	0.94	<b>0.94</b>	0.94	0.15	0.95	<b>0.95</b>	0.94	0.15	0.94	<b>0.94</b>	0.94
0.23	0.95	0.94	0.94	0.23	0.95	0.94	0.94	0.23	0.94	0.94	0.94

Table 11: Shares across Depreciation Rates

This table reports the model-implied input-shares ( $\mu$ ) for all firms, low and high skill using the baseline model specification for the different assumptions on depreciation rates. Across columns we vary the depreciation rate of brand capital from 10% to 30%, and across rows we vary the depreciation rate of knowledge capital, from 7% to 23%. The center estimate in bold is equivalent to the specification in the main draft.

	All Firms			Low Skill			High Skill				
	0.10	0.20	0.30	0.10	0.20	0.30	0.10	0.20	0.30		
<b>Firm-value decomposition - Aggregate (in %)</b>											
	$\mu^P : Physical$				$\mu^P : Physical$				$\mu^P : Physical$		
0.07	30.27	30.73	30.30	0.07	38.47	38.07	38.94	0.07	30.45	30.92	30.43
0.15	30.64	<b>30.36</b>	30.46	0.15	38.49	<b>40.16</b>	40.39	0.15	29.77	<b>29.91</b>	30.18
0.23	30.27	29.62	29.70	0.23	37.92	39.08	39.79	0.23	29.15	29.50	29.89
	$\mu^L : Labor$				$\mu^L : Labor$				$\mu^L : Labor$		
0.07	20.34	20.46	20.79	0.07	13.07	14.19	14.05	0.07	18.18	18.50	18.72
0.15	21.90	<b>22.53</b>	22.54	0.15	13.35	<b>14.33</b>	14.21	0.15	20.42	<b>20.85</b>	20.64
0.23	22.54	23.30	23.30	0.23	13.53	14.64	14.68	0.23	21.47	21.51	21.33
	$\mu^K : Knowledge$				$\mu^K : Knowledge$				$\mu^K : Knowledge$		
0.07	40.18	40.33	40.47	0.07	23.30	22.97	22.38	0.07	44.75	44.84	44.95
0.15	38.00	<b>38.28</b>	38.26	0.15	21.86	<b>20.34</b>	20.67	0.15	43.05	<b>43.23</b>	43.14
0.23	37.02	37.51	37.54	0.23	20.87	20.00	20.16	0.23	42.32	42.47	42.51
	$\mu^B : Brand$				$\mu^B : Brand$				$\mu^B : Brand$		
0.07	9.21	8.47	8.44	0.07	25.16	24.77	24.63	0.07	6.62	5.74	5.89
0.15	9.46	<b>8.83</b>	8.73	0.15	26.30	25.17	24.74	0.15	6.76	6.02	6.04
0.23	10.17	9.58	9.46	0.23	27.68	26.28	25.37	0.23	7.07	6.52	6.27
<b>Firm-value decomposition - Average (in %)</b>											
	$\mu^P : Physical$				$\mu^P : Physical$				$\mu^P : Physical$		
0.07	22.39	22.60	22.08	0.07	40.88	40.27	41.29	0.07	22.03	22.26	21.77
0.15	22.43	<b>21.85</b>	21.90	0.15	41.18	<b>42.64</b>	42.83	0.15	21.01	<b>20.91</b>	21.13
0.23	22.02	20.98	21.01	0.23	40.64	41.58	42.38	0.23	20.20	20.32	20.61
	$\mu^L : Labor$				$\mu^L : Labor$				$\mu^L : Labor$		
0.07	24.02	23.99	24.38	0.07	16.48	17.94	17.76	0.07	21.38	21.66	21.88
0.15	26.01	<b>26.61</b>	26.62	0.15	17.00	<b>18.14</b>	17.95	0.15	23.94	<b>24.32</b>	24.10
0.23	26.98	27.75	27.76	0.23	17.39	18.67	18.63	0.23	25.23	25.22	25.01
	$\mu^K : Knowledge$				$\mu^K : Knowledge$				$\mu^K : Knowledge$		
0.07	49.13	48.90	49.03	0.07	25.73	25.23	24.66	0.07	53.05	52.79	52.91
0.15	47.00	<b>46.84</b>	46.81	0.15	23.92	<b>22.19</b>	22.64	0.15	51.52	<b>51.36</b>	51.28
0.23	46.01	46.15	46.15	0.23	23.00	21.86	21.96	0.23	50.86	50.76	50.77
	$\mu^B : Brand$				$\mu^B : Brand$				$\mu^B : Brand$		
0.07	4.45	4.51	4.52	0.07	16.91	16.56	16.30	0.07	3.54	3.29	3.45
0.15	4.56	<b>4.70</b>	4.67	0.15	17.89	<b>17.03</b>	16.58	0.15	3.53	<b>3.41</b>	3.49
0.23	4.99	5.12	5.07	0.23	18.98	17.89	17.04	0.23	3.71	3.70	3.62



Table 12: Realized Adjustment Costs across Depreciation Rates

This table reports the estimated realized adjustment costs ( $CX/Y$ ) for all firms, low and high skill using the baseline model specification for the different assumptions on depreciation rates. Across columns we vary the depreciation rate of brand capital from 10% to 30%, and across rows we vary the depreciation rate of knowledge capital, from 7% to 23%. The center estimate in bold is equivalent to the specification in the main draft.

	All Firms			Low Skill			High Skill				
	0.10	0.20	0.30	0.10	0.20	0.30	0.10	0.20	0.30		
%	$CP/Y$			$CP/Y$			$CP/Y$				
0.07	1.17	1.21	1.08	0.07	1.10	0.96	1.04	0.07	2.00	2.04	1.89
0.15	1.03	<b>0.90</b>	0.92	0.15	1.09	<b>1.22</b>	1.22	0.15	1.55	<b>1.50</b>	1.56
0.23	0.88	0.67	0.67	0.23	1.04	1.10	1.15	0.23	1.26	1.30	1.37
	$CL/Y$			$CL/Y$			$CL/Y$				
0.07	6.12	6.09	6.16	0.07	2.44	2.60	2.55	0.07	6.25	6.29	6.34
0.15	6.37	<b>6.46</b>	6.45	0.15	2.48	<b>2.61</b>	2.58	0.15	6.70	<b>6.77</b>	6.70
0.23	6.46	6.58	6.56	0.23	2.53	2.68	2.66	0.23	6.92	6.87	6.81
	$CK/Y$			$CK/Y$			$CK/Y$				
0.07	6.37	6.28	6.26	0.07	1.63	1.55	1.48	0.07	8.38	8.26	8.25
0.15	10.16	<b>10.05</b>	10.01	0.15	2.62	<b>2.35</b>	2.38	0.15	13.42	<b>13.28</b>	13.24
0.23	14.24	14.17	14.14	0.23	3.63	3.37	3.36	0.23	18.82	18.70	18.68
	$CB/Y$			$CB/Y$			$CB/Y$				
0.07	0.09	0.49	0.92	0.07	0.72	1.66	2.63	0.07	0.04	0.31	0.70
0.15	0.08	<b>0.49</b>	0.91	0.15	0.78	<b>1.69</b>	2.64	0.15	0.02	<b>0.30</b>	0.67
0.23	0.13	0.56	1.01	0.23	0.86	1.80	2.72	0.23	0.04	0.36	0.70

Table 13: Adjustment Costs at 10% Investment

This table reports the estimated adjustment costs ( $CX/Y$ ) for a 10% investment rate for all firms, low and high skill using the baseline model specification for the different assumptions on depreciation rates. Across columns we vary the depreciation rate of brand capital from 10% to 30%, and across rows we vary the depreciation rate of knowledge capital, from 7% to 23%. The center estimate in bold is equivalent to the specification in the main draft.

	All Firms			Low Skill			High Skill				
	0.10	0.20	0.30	0.10	0.20	0.30	0.10	0.20	0.30		
	$CP/Y$			$CP/Y$			$CP/Y$				
0.07	0.20	0.20	0.18	0.07	0.43	0.37	0.40	0.07	0.28	0.28	0.26
0.15	0.17	<b>0.15</b>	0.15	0.15	0.42	<b>0.47</b>	0.47	0.15	0.21	<b>0.21</b>	0.22
0.23	0.15	0.11	0.11	0.23	0.40	0.43	0.45	0.23	0.17	0.18	0.19
	$CL/Y$			$CL/Y$			$CL/Y$				
0.07	1.81	1.80	1.83	0.07	0.89	0.95	0.94	0.07	1.77	1.78	1.79
0.15	1.89	<b>1.91</b>	1.91	0.15	0.91	<b>0.96</b>	0.95	0.15	1.90	<b>1.92</b>	1.90
0.23	1.92	1.95	1.94	0.23	0.93	0.98	0.98	0.23	1.96	1.94	1.93
	$CK/Y$			$CK/Y$			$CK/Y$				
0.07	1.56	1.54	1.53	0.07	0.79	0.75	0.72	0.07	1.81	1.78	1.78
0.15	1.07	<b>1.06</b>	1.06	0.15	0.53	<b>0.47</b>	0.48	0.15	1.30	<b>1.29</b>	1.29
0.23	0.85	0.85	0.84	0.23	0.43	0.40	0.40	0.23	1.03	1.02	1.02
	$CB/Y$			$CB/Y$			$CB/Y$				
0.07	0.04	0.08	0.09	0.07	0.35	0.29	0.22	0.07	0.02	0.05	0.05
0.15	0.03	<b>0.08</b>	0.09	0.15	0.38	<b>0.30</b>	0.22	0.15	0.01	<b>0.05</b>	0.05
0.23	0.05	0.09	0.10	0.23	0.42	0.32	0.23	0.23	0.01	0.06	0.05

## 4 Additional Results

### 4.1 Heterogeneity in the Adjustment Costs and Shares

560 In the main text, we summarize the properties of the input-shares in the economy using the aggregate and average measures. Here, we add to that analysis by investigating the degree of input-share heterogeneity in the firm-level data. Figure 2 shows the box plot (across all years) of the firm-value input-shares for the low- and high-skill industries. This figure reveals that there is substantial heterogeneity in input-shares both in low- and high-skill industries. For example, for physical  
565 capital, the 25th and 75th percentile in low-skill industries are around 22% and 50%, respectively, and in high-skill industries they are around 10% and 30%, respectively. For labor, the 25th and 75th

percentile in low-skill industries are around 5% and 25%, respectively, and in high-skill industries they are around 10% and 40%, respectively. For knowledge capital, the 25th and 75th percentile in low-skill industries are around 10% and 30%, respectively, and in high-skill industries they are  
570 around 25% and 70%, respectively. Finally, for brand capital, the 25th and 75th percentile in low-skill industries are around 5% and 30%, respectively, but in high skill industries, the mass of the share is concentrated at very low levels, all below 10%. Thus, the relatively low share of brand capital for firm value in high-skill industries is a consistent feature across all firms in these industries.

To evaluate the degree of firm-level heterogeneity in the realized adjustment costs of each input  
575 in the data, Figure 3 shows the box plot of the ratios in the low- and high-skill industries. The box plot of the realized adjustment costs in each industry shown in Figure 3 reveals that there is substantial variation in the realized input adjustment costs across firms. As expected, given the strong link between input-shares and adjustment costs, the pattern in the box-plots of the realized firm-level realized adjustment costs across industries and inputs seems to mimic the pattern and  
580 the large variation in firm-level shares of each input reported in Figure 2.

## 4.2 Estimation Across the Fama and French Industry Classifications

Tables 14 to 18 report the results of the benchmark model estimation (quadratic costs with 40 portfolios) for the Fama-French industries. Note that we perform the estimation separately across each one of the seven Fama-French industries based on the Fama-French ten industry classification  
585 (we exclude three industries due to data availability), thus allowing us to check the robustness of the findings to the industry classification. We use fewer industries here than in the previous subsection 3.2.2 because we need a sufficient number of observations over all time periods to construct the portfolios.

Figure 2: Distribution of Input Market Value Shares

This figure shows the distribution (box plot) of the estimated firm-level input shares ( $\mu$ ) in high- and low-skill industries, using the parameter estimates reported in Table 3 in the main text, columns (2) and (3), to obtain the input-shares. In each box, the central mark is the median, the edges of the box are the 25th and 75th percentiles. The whiskers extend to the most extreme data points the algorithm considers not to be outliers.

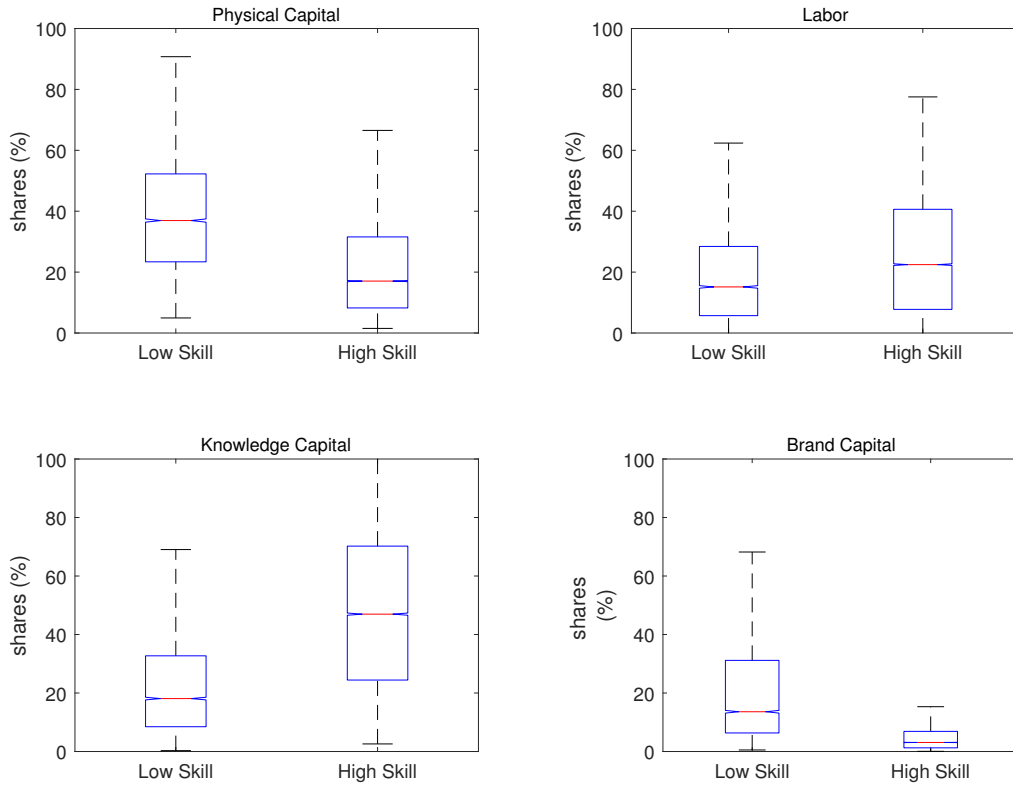


Figure 3: Distribution of Realized Input Adjustment Costs

This figure shows the distribution (box plot) of the estimated firm-level adjustment costs as a fraction of firms' annual sales ( $CX/Y$ ) in high- and low-skill industries, using the parameter estimates reported in Table 3 in the main text, columns (2) and (3), to calculate the adjustment costs. In each box, the central mark is the median, the edges of the box are the 25th and 75th percentiles. The whiskers extend to the most extreme data points the algorithm considers not to be outliers.

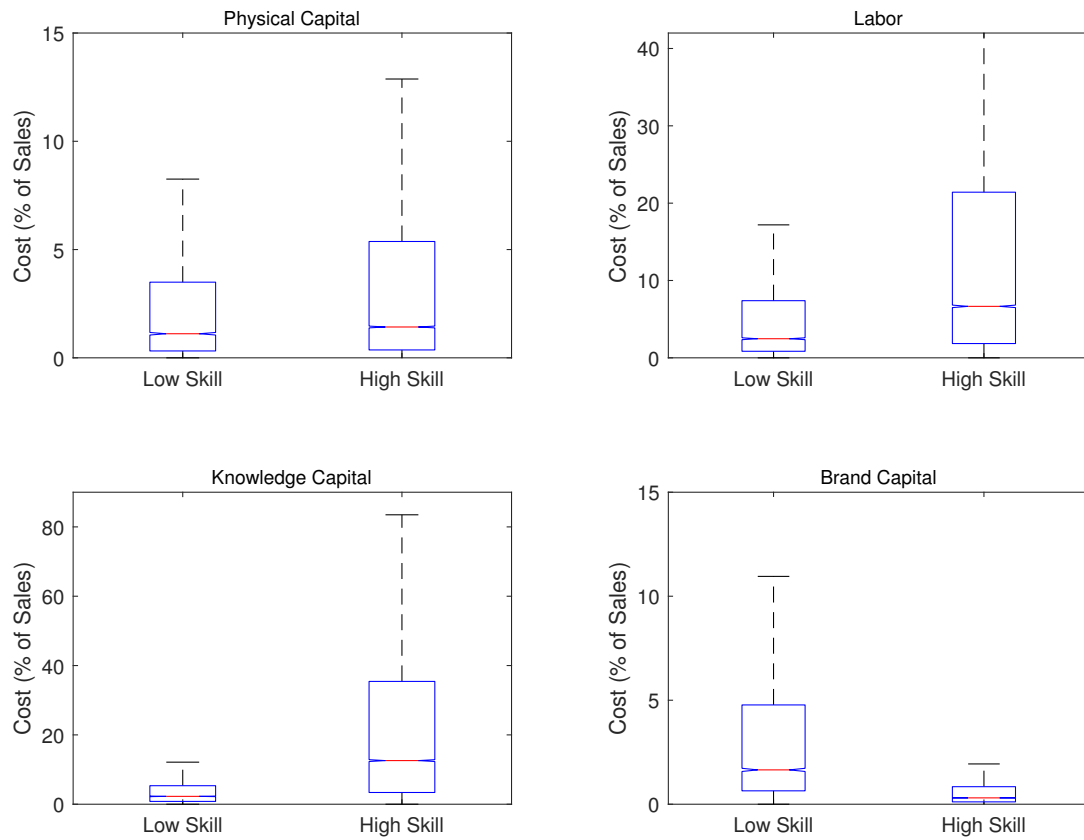


Table 14: Descriptive Statistics Across Fama-French Industries

This table reports the time-series average of the cross-sectional median, and the standard-deviation of selected characteristics of the firm-level data.  $VR_{it}$  is the firm's valuation ratio.  $I_{it}^P/K_{it}^P$  is the investment rate in physical capital,  $H_{it}/L_{it}$  is the investment rate in labor stock (hiring rate),  $I_{it}^K/K_{it}^K$  is the investment rate in knowledge capital and  $I_{it}^B/K_{it}^B$  is the investment rate in brand capital. We also present the stock of each input (physical capital, labor, knowledge capital and brand capital) relative to the sum of the three capital inputs ( $A_{it}$ ) and relative to sales ( $Y_{it}$ ). The sample is annual data from 1975 to 2016.

	Average										S.D.			
	NoDur	Durbl	Manuf	HiTech	Shops	Hlth	Other	NoDur	Durbl	Manuf	HiTech	Shops	Hlth	Other
$VR_{it}$	1.38	1.75	1.59	2.13	2.14	2.57	1.95	2.56	2.89	2.58	3.96	4.27	4.11	3.67
	Valuation ratios													
	Investment/hiring rates													
$I_{it}^P/K_{it}^P$	0.14	0.18	0.14	0.35	0.24	0.27	0.25	0.38	0.44	0.38	0.71	0.64	0.69	0.68
$H_{it}/L_{it}$	0.14	0.13	0.13	0.18	0.25	0.19	0.29	0.20	0.22	0.21	0.28	0.29	0.26	0.28
$I_{it}^K/K_{it}^K$	0.21	0.26	0.21	0.37	0.17	0.24	0.21	0.16	0.18	0.15	0.23	0.24	0.2	0.23
$I_{it}^B/K_{it}^B$	0.24	0.23	0.23	0.26	0.25	0.27	0.26	0.17	0.16	0.15	0.23	0.21	0.25	0.21
	Scaled capital and labor ratios													
$K_{it}^P/A_{it}$	0.56	0.57	0.61	0.3	0.55	0.28	0.60	0.24	0.21	0.22	0.23	0.28	0.23	0.29
$(W_{it-1}L_{it})/A_{it}$	0.44	0.86	0.69	0.62	0.78	0.49	0.59	0.52	0.87	0.57	1.15	2.36	0.85	5.31
$K_{it}^K/A_{it}$	0.10	0.22	0.22	0.53	0.12	0.56	0.16	0.16	0.19	0.19	0.24	0.23	0.26	0.25
$K_{it}^B/A_{it}$	0.27	0.14	0.10	0.09	0.22	0.07	0.10	0.2	0.16	0.16	0.13	0.21	0.14	0.17
	Capital and labor relative to sales													
$K_{it}^P/Y_{it}$	0.23	0.21	0.25	0.15	0.11	0.22	0.23	0.22	0.2	0.25	0.24	0.22	0.45	0.44
$(W_{it-1}L_{it})/Y_{it}$	0.21	0.34	0.32	0.37	0.19	0.4	0.31	0.25	0.29	1.04	27.15	2.93	31.14	0.88
$K_{it}^K/Y_{it}$	0.04	0.09	0.09	0.26	0.02	0.38	0.06	0.53	0.49	0.49	0.93	0.68	2.04	0.72
$K_{it}^B/Y_{it}$	0.12	0.05	0.05	0.05	0.06	0.06	0.04	0.21	0.12	0.16	0.15	0.2	0.24	0.17

Table 15: Parameter Estimates and Model Fit Across Fama-French Industries

This table reports estimation results and measures of model fit. The columns show the values for benchmark estimates for the Fama-French industries. The estimation uses forty portfolios based on the lagged investment/hiring (ten based on each type of input) for all industries except for the industry “shops” which, due to data limitations, is performed using 5 portfolios of each. The estimation is done using the cross-sectional average aggregation method and LS methodology. The first panel reports the estimation results.  $\theta_P$ ,  $\theta_L$ ,  $\theta_K$  and  $\theta_B$  are respectively, the physical capital, labor, knowledge capital and brand capital slope adjustment costs parameters. s.e. stands for Newey-West standard errors with three lags. The second panel reports measures of fit, m.a.e./|VR| is the mean absolute valuation error scaled by the absolute value of the ratio. The sample is annual data from 1975 to 2016.

	NoDur (1)	Durbl (2)	Manuf (3)	HiTech (4)	Shops (5)	Hlth (6)	Other (7)
Parameter estimates							
$\theta_P$	6.86	2.81	3.84	4.12	4.51	8.41	5.29
s.e.	[0.97]	[0.66]	[0.89]	[0.90]	[1.24]	[1.01]	[1.08]
$\theta_L$	3.04	4.53	8.10	8.55	6.29	7.00	3.96
s.e.	[0.85]	[0.55]	[0.70]	[0.64]	[0.85]	[0.72]	[0.51]
$\theta_K$	18.00	24.63	14.68	10.55	21.05	16.51	13.12
s.e.	[2.82]	[1.38]	[1.69]	[0.75]	[3.47]	[0.81]	[1.66]
$\theta_B$	10.35	1.66	10.73	1.84	16.07	4.30	8.74
s.e.	[1.41]	[1.62]	[1.95]	[3.62]	[2.82]	[1.66]	[2.40]
Model fit							
XS- $R^2$	0.45	0.83	0.86	0.92	0.90	0.93	0.84
TS- $R^2$	0.12	0.38	0.39	0.52	0.38	0.47	0.23
m.a.e./VR	0.47	0.35	0.29	0.26	0.38	0.27	0.50

Table 16: Estimated Input-Shares Across Fama-French Industries

This table reports the fraction of firm value that is attributed to each input ( $\mu$ , input-shares) based on their book and market values for the Fama-French industries. The book-value decomposition is done by setting all the adjustment costs to zero. We use the parameter estimates reported in Table 15 to calculate the market value decomposition. We report both the aggregate and the average input-share decomposition. The table shows the time series averages between 1975 to 2016.

	NoDur (1)	Durbl (2)	Manuf (3)	HiTech (4)	Shops (5)	Hlth (6)	Other (7)
Book value decomposition - Aggregate (in %)							
$\bar{\mu}^P$ : Physical capital	62.22	71.28	70.68	53.89	77.86	45.95	77.21
$\bar{\mu}^L$ : Labor	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\bar{\mu}^K$ : Knowledge capital	8.00	18.56	18.03	36.74	4.05	38.84	11.41
$\bar{\mu}^B$ : Brand capital	29.78	10.16	11.29	9.36	18.08	15.22	11.38
Book value decomposition - Average (in %)							
$\bar{\mu}^P$ : Physical capital	65.60	69.71	74.58	45.25	69.72	44.43	71.52
$\bar{\mu}^L$ : Labor	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\bar{\mu}^K$ : Knowledge capital	9.06	19.22	17.33	47.28	10.50	48.25	17.82
$\bar{\mu}^B$ : Brand capital	25.34	11.08	8.09	7.47	19.78	7.32	10.66
Market value decomposition - Aggregate (in %)							
$\bar{\mu}^P$ : Physical capital	39.96	34.29	39.41	27.95	38.66	28.83	50.92
$\bar{\mu}^L$ : Labor	4.96	5.36	15.11	19.23	25.47	8.35	15.1
$\bar{\mu}^K$ : Knowledge capital	15.09	55.41	29.88	48.94	8.52	52.66	18.1
$\bar{\mu}^B$ : Brand capital	39.99	4.94	15.6	3.88	27.35	10.17	15.88
Market value decomposition - Average (in %)							
$\bar{\mu}^P$ : Physical capital	43.84	32.84	39.93	21.52	33.05	25.87	49.78
$\bar{\mu}^L$ : Labor	4.74	11.83	19.81	19.88	25.97	11.71	18.41
$\bar{\mu}^K$ : Knowledge capital	18.08	50.05	28.82	55.8	13.47	58.43	20.65
$\bar{\mu}^B$ : Brand capital	33.34	5.28	11.44	2.81	27.51	3.99	11.16



Table 17: Estimated Input-Shares Over Time and Across Fama-French Industries

This table shows the input-shares ( $\mu$ ) attributed to each input across different decades. The calculations are done using the parameter estimates reported in Table 15 for each industry. The input-shares are computed at the aggregate level.

	70s	80s	90s	00s	10s	70s	80s	90s	00s	10s
	Non Durables					Shops				
$\bar{\mu}^P$ : Physical capital	49.06	44.28	34.57	36.77	39.62	51.1043	47.2161	40.8708	29.5818	26.4864
$\bar{\mu}^L$ : Labor	6.47	5.10	4.28	4.81	4.87	22.2878	19.3166	23.7452	27.5963	37.6478
$\bar{\mu}^K$ : Knowledge capital	9.57	10.15	14.37	19.80	20.29	3.9825	12.2312	7.323	9.9317	5.7086
$\bar{\mu}^B$ : Brand capital	34.90	40.46	46.79	38.61	35.22	22.6254	21.2361	28.061	32.8902	30.1573
	Durables					Healthcare				
$\bar{\mu}^P$ : Physical capital	35.38	37.57	33.24	34.00	31.63	37.41	34.33	27.21	24.42	23.70
$\bar{\mu}^L$ : Labor	9.06	4.80	5.54	4.12	5.47	11.54	8.66	8.71	7.31	6.71
$\bar{\mu}^K$ : Knowledge capital	51.27	53.12	55.80	56.87	57.77	34.56	42.37	55.24	61.57	63.52
$\bar{\mu}^B$ : Brand capital	4.29	4.51	5.42	5.02	5.13	16.49	14.64	8.83	6.71	6.08
	Manufacturing					Other				
$\bar{\mu}^P$ : Physical capital	49.92	46.02	37.62	33.70	33.34	73.85	52.31	45.93	48.05	42.88
$\bar{\mu}^L$ : Labor	13.13	10.08	15.82	18.30	18.16	13.30	17.81	20.15	11.33	11.22
$\bar{\mu}^K$ : Knowledge capital	23.29	29.56	30.92	31.06	31.83	4.75	13.42	18.27	25.51	22.83
$\bar{\mu}^B$ : Brand capital	13.65	14.35	15.64	16.94	16.67	8.10	16.46	15.65	15.11	23.07
	HiTech									
$\bar{\mu}^P$ : Physical capital	38.17	34.96	26.25	22.49	20.78					
$\bar{\mu}^L$ : Labor	23.17	17.38	17.80	19.71	20.51					
$\bar{\mu}^K$ : Knowledge capital	34.54	43.42	52.01	54.13	55.28					
$\bar{\mu}^B$ : Brand capital	4.12	4.24	3.93	3.67	3.43					

Table 18: Estimated Adjustment Costs Across Fama-French Industries

The first panel of this table displays the average skill index within each Fama-French Industry. The skill index ranges from 1 (lowest skill level) to 10 (highest skill level) and the data is from Belo et al. (2017). The second panel evaluates the adjustment costs using the parameter estimates reported in Table 15 and evaluated at 10% investment and hiring rates as a proportion of the respective (average) median input stock-to-sales in Table 14.

	NoDur (1)	Durbl (2)	Manuf (3)	HiTech (4)	Shops (5)	Hlth (6)	Other (7)
Labor skill index							
	3.42	5.53	6.69	9.70	4.87	8.38	6.30
Adjustment costs evaluated at 10% investment/hiring							
<i>CP/Y</i> : Physical capital	1.58	0.59	0.96	0.62	0.50	1.85	1.22
<i>CL/Y</i> : Labor	0.64	1.54	2.59	3.16	1.20	2.80	1.23
<i>CK/Y</i> : Knowledge capital	0.72	2.22	1.32	2.74	0.42	6.27	0.79
<i>CB/Y</i> : Brand capital	1.24	0.08	0.54	0.09	0.96	0.26	0.35
Realized adjustment costs (in %)							
<i>CP/Y</i> : Physical capital	1.93	1.1	1.15	4.27	1.82	7.18	5.91
<i>CL/Y</i> : Labor	0.74	1.71	2.84	7.04	4.45	6.07	5.92
<i>CK/Y</i> : Knowledge capital	2.24	7.14	3.06	20.54	1.61	20.45	2.94
<i>CB/Y</i> : Brand capital	3.56	0.22	1.24	0.29	3.27	1.06	1.43

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