Decomposing Firm Value*

Frederico Belo†
INSEAD and NBER

Vito D. Gala‡
PIMCO

Juliana Salomao§
University of Minnesota

Maria Ana Vitorino¶
INSEAD

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Abstract

What are the economic determinants of a firm's market value? We answer this question through the lens of a generalized neoclassical model of investment with quasi-fixed labor and three heterogeneous capital inputs. We estimate the structural model using firm-level data on U.S. firms and find that, on average, installed labor force accounts for 14% to 22% of firms' market value, physical capital accounts for 30% to 40%, knowledge capital accounts for 20% to 43%, and brand capital accounts for 6% to 25%. Our analysis provides direct empirical evidence for the importance of labor and intangible capital inputs for understanding firm value.

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†Corresponding author: Frederico Belo, INSEAD, Boulevard de Constance 77300, Fontainebleau, France; telephone: +33 1 60 72 45 24. E-mail: frederico.belo@insead.edu.

‡vito.gala@gmail.com.

§jsalomao@umn.edu.

¶maria-ana.vitorino@insead.edu.

1 Introduction

Understanding the economic determinants of a firm's market value is an important question that has attracted substantial research in finance and economics. We address this question through the lens of a generalized neoclassical model of investment with four different types of quasi-fixed inputs: physical capital (machines and plants), labor (workers), and two types of intangible capital, namely knowledge capital (accumulated investment in innovation activities), and brand capital (accumulated investment in improving brand awareness). This rich model of the firm incorporates the evidence from Hall (2001), McGrattan and Prescott (2000), and Merz and Yashiv (2007) that intangible capital and installed labor force are important components of firms' stock market values at the aggregate level. Through structural estimation, and using data for a large cross section of publicly traded firms in the U.S. economy, we use the model to quantify the relative importance of the various capital inputs and labor for understanding firms' market values, both across industries and over time.

In the model, changing the quantity of the capital inputs and labor is costly, which we capture through standard adjustment cost functions. For physical and intangible capital inputs these costs include, for example, planning and installation costs, and costs related with production being temporarily interrupted. For labor, these costs include the costs of hiring and firing workers, as well as the costs of training new workers. The firm's equilibrium market value depends on the shadow price and the quantity of each installed input, and the shadow prices can be inferred from investment and hiring data through the specification of an adjustment costs function. If the operating profit function and the adjustment costs function are both homogeneous of degree one (the Hayashi 1982 conditions), the market value of each input is the product of the input's shadow price and the corresponding stock variable. The total market value of the firm is then the sum of the market value of all the inputs, and this additive property allows us to compute the contribution of each input for firm value in a straightforward manner.

To take the model to the data, we need to measure the firm-level stocks of each capital input and labor. For physical capital and labor, the data is readily available from the firm's 10-K reports. For knowledge capital and brand capital, the capital stock data is not readily available given its intangible nature. Following previous studies, we construct firm-level measures of knowledge capital stock and brand capital stock from firm-level accounting data on research and development (R&D) expenses, and data on advertising expenses, respectively. Accordingly, we interpret R&D expenditures as a firm's investment to generate new (or improve current) ideas. Similarly, we interpret advertising expenses as a firm's investment to enhance the value of brand names and increase brand awareness. We accumulate these expenditures using the perpetual inventory method to obtain the capital stocks for knowledge capital and brand capital.

We estimate the model by minimizing the distance between the observed and the model-implied valuation ratios (market value of equity plus net debt-to-book value of capital stocks). To abstract from idiosyncratic shock responses that add noise to the firm-level data, we estimate the model using portfolio-level moments as in Belo, Xue, and Zhang (2013) (henceforth BXZ), who in turn follow the original estimation approach in Liu, Whited, and Zhang (2009) (henceforth LWZ). We consider portfolios sorted on proxies of the firm's lagged market value of the capital and labor inputs. This sorting generates a large dispersion in the explanatory variables which helps the identification of the model parameters. We perform the estimation in a pooled sample that includes all firms in the economy, and also separately within different industries. Following Belo et al. (2017), we split the sample into low, and high labor-skill industries (henceforth low-, and high-skill industries), based on the industry-level average fraction of workers in that industry that are classified as high-skill workers. To a first approximation these industries correspond to low-, and high-tech sectors of the economy.

We modify the portfolio-level estimation procedure in BXZ and LWZ in two important ways. First, to estimate the model parameters, we target cross-sectional portfolio-level moments rather than a portfolio-level aggregate valuation ratio. This modification is important to recover the true firm-level structural parameters since the procedure in BXZ and LWZ is subject to an aggregation bias. Second, we match the realized time series of the portfolio-level valuation ratios as close as possible, as opposed to just the time series average of the valuation ratios as in BXZ and LWZ. This modification is important in the context of our analysis because the contribution of some of the inputs for firm value changes over time.

Our main empirical findings can be summarized as follows. In the pooled sample, the model performs well in explaining both the time-series and the cross-sectional variation of the valuation ratios across portfolios, with a time-series R^2 of 61% and a cross-sectional R^2 of 94%. The model fit is particularly good in high-skill industries, with a time-series R^2 of 60%, whereas the model fit in the low-skill industries is more modest, with a time-series R^2 of 38%. The cross-sectional fit is good in both industries, with cross-sectional R^2s above 94%.

To help understand the good fit of the model and the relative importance of each capital input and labor for firm's valuation, we estimate restricted versions of the model using subsets of the inputs. Consistent with BXZ, the standard one-physical-capital input model does a reasonable job explaining the cross-sectional variation in the average valuation ratio across portfolios with a cross-sectional R^2 of 50% in low-skill industries, and of 75% in high-skill industries. But the one-physical-capital input model fails to explain the time-series variation in the valuation ratios, with a time-series R^2 of effectively 0% in low-skill industries, and 21% in high-skill industries. Thus, we conclude that the benefit of incorporating additional quasi-fixed inputs in the neoclassical investment model comes primarily from improving the model's ability to capture the time-series variation in firms' valuation ratios.

Comparing across alternative model specifications, we find that the contribution of each input for the improvement of the model fit relative to the one-physical-capital input model varies across industries. Adding labor and, especially, knowledge capital, to the one-physical-capital input model has a first-order impact on the quality of the model fit in both industries (and especially in high-skill industries), whereas adding brand capital has a significantly impact on the quality of the model fit in low-skill industries only.

More important, the parameter estimates allow us to quantitatively evaluate the relative contribution of each input for firm value. In the pooled sample, and depending on how the data is aggregated for reporting purposes, we find that, on average, physical capital accounts for 22% to 30% of firms' market value, installed labor force accounts for 23% to 27%, knowledge capital accounts for 38% to 47%, and brand capital accounts for the remaining 5% to 9%. Thus, on average, the non-physical capital inputs account for the majority, between 70% and 80%, of firms' market value.

The relative importance of the capital and labor inputs for firms' market value varies substantially across industries. On average, the contribution of physical capital is higher in low-skill industries (about 40% to 43% of a firm's market value) than in high-skill industries (about 21% to 30% of a firm's market value). This result suggests that the standard one-physical-capital input model is a more appropriate model of the firm in low-skill industries than in high-skill industries. Related, we show that the contribution of labor and knowledge capital for firm value increases with the average labor-skill level of the industry. In low-skill industries, the contribution of labor and knowledge capital is on average only 14% to 18% and 20% to 22%, respectively, whereas in high-skill industries the contribution is 21% to 24% and 43% to 51%, respectively. This result suggests that adding labor and knowledge capital to the one-physical-capital input model is especially important for understanding the valuation of firms in high-skill industries.

In addition, we find that the contribution of brand capital for firm value decreases with the average labor-skill level of the industry. Brand capital appears to be very important in low-skill industries, where it accounts on average for about 25% of firm value, but not so much in high-skill industries where it accounts on average for about 6% of firm value. Thus, our estimates show that, even though intangible capital is an important component of the firm's market value across all industries, the type of intangible capital (knowledge or brand capital) that matters the most for firm value varies across industries. This result highlights the importance of considering heterogeneous measures of intangible capital in empirical work.

The estimated contributions of each input for firm value also vary substantially over time. In the last four decades, in the pooled sample, the importance of knowledge capital increased significantly from 25% in the 1970s to 45% in the 2010s. The increased importance of knowledge capital crowded out the importance of physical capital. The importance of physical capital for firm value has significantly decreased from 43% in the 1970s to 23% in the 2010s. The contribution of labor and brand capital for firm value do not exhibit an obvious trend as their values have remained relatively constant over the sample period. The decline in the contribution of physical capital for firm value and the increase in the contribution of knowledge capital for firm value is present in both low- and high-skill industries (although the change is more pronounced in high-skill industries). This result suggests that the trends in the relative contribution of the inputs observed in the pooled sample is not due to changes in the industry composition in the U.S. economy but rather seems to be driven by a trend in the overall economy.

What explains the estimated firm-value decomposition? As noted, the value of each input is determined by the product of the input's shadow price and its book-value. In equilibrium, the shadow price of each input equals its marginal investment cost, which depends on investment and hiring and on the adjustment cost parameters. In particular, all else equal, the more costly it is to adjust inputs to changing economic conditions, the more valuable the existing stock of the inputs. Thus, understanding the adjustment costs estimates is important for understanding the relative contribution of each input for firm value.

For example, in the case of labor, without labor adjustment costs, the shadow price of labor is zero because firms do not sell nor buy workers as they do with capital goods, but the market value of labor might be different from zero when it is costly to adjust the labor force (due to a nonzero labor shadow price). This is because, in equilibrium, firms extract rents from labor as a compensation for the costs of adjusting the labor force in the future. The same logic applies to the other capital inputs. The difference relative to labor is that the value of these inputs is not zero, even without adjustment costs, because firms can sell (or buy) capital.

Our estimates show that adjusting the four inputs in response to changing economic conditions is fairly costly, especially labor and knowledge capital. Using the estimates from the pooled sample, we find that a firm's annual labor adjustment costs represent on average about 6.5% of total annual sales, consistent with estimates in Merz and Yashiv (2007). In addition, knowledge capital adjustment costs represent on average about 10% of total annual sales. These figures are significantly higher than the physical capital and brand capital average adjustment costs of about 0.9% and 0.5% of total annual sales, respectively.

The estimated size of the adjustment costs of the different capital and labor inputs varies substantially across industries, and, except for brand capital, are higher in high-skill industries. The fraction of sales lost due to labor adjustment costs is on average 2.6% in low-skill industries and 6.8% in high-skill industries. Thus, consistent with previous studies, we find that it is more costly to replace high-skill than low-skill workers (see discussion in related literature section below). Similarly, the fraction of sales lost due to knowledge capital adjustment costs is on average 2.4% in low-skill industries and 13.3% in high-skill industries. The difference in the size of adjustment costs across industries is less pronounced for physical capital. The fraction of sales lost due to physical capital adjustment costs is on average 1.2% in low-skill industries and 1.5% in high-skill industries. The positive relationship between adjustment costs and average labor-skill of the industry is reversed for brand capital. The fraction of sales lost due to brand capital adjustment costs is on average 1.7% in low-skill industries and 0.3% in high-skill industries.

Finally, we provide a series of robustness checks to establish the importance of non-physical capital inputs for firm value. We re-estimate the model across several perturbations of the empirical procedures and for different data samples. First, we estimate the model assuming a more general adjustment costs function that allows adjustment costs to be asymmetric. The estimated contributions of each input for firm value and the adjustment costs under this specification are very similar to ones from the benchmark model. Second, we consider a larger number of portfolios as test assets and different portfolio sorts. Again, the contribution of each input for firm value and the estimated adjustment costs are robust. Third, we also investigate the robustness of the findings using an alternative industry classification. We estimate the adjustment costs parameters separately for each Fama-French industry, and provide industry-specific values. While the decomposition across inputs is heterogeneous, physical capital still accounts for less than 50% of the firm value across industries. Fourth, using firm- (instead of portfolio-) level estimation, reveals that, even in the presence of noise in firm-level data, the estimated contribution of the non-physical capital inputs for firm value is still substantial, approximately 60%. Fifth, we re-estimate a restricted version of the model without knowledge capital using the sub-sample of firms excluded from the main sample due to missing (or always zero) R&D expenses data, and find that, similar to the main model results, the non-physical capital inputs (labor and brand capital) account for a large fraction (more than 38%) of firm's value in this alternative sample.

As an application of our findings, our analysis provides new insights about which types of inputs contribute the most to the difference in returns between growth stocks (which have high valuation ratios) and value stocks (which have low valuation ratios), that is, the value premium in financial markets. Our results show that, while the contribution of labor for firm value is higher for growth firms than for value firms (29% versus 19%), the contribution of physical capital is lower for growth firms than for value firms (24% versus 35%). Regarding the intangible capitals, growth firms derive more value from knowledge capital than from brand capital (40% versus 36%)

while value firms derive more value from brand capital than from knowledge capital (10% versus 6%). These results are consistent with the notation that value firms derive more value from current assets in the form of physical capital and established brand capital, while growth firms derive more value from growth opportunities in the form of labor and knowledge capital. Thus, frictions in labor, knowledge capital, and brand capital, in addition to frictions in physical capital, seem to be important for understanding the value premium in financial markets.

Taken together, our results provide direct empirical evidence supporting models with multiple capital inputs as main sources of firm value, and show the importance of non-physical capital inputs for firm value.

The rest of the paper proceeds as follows. Section 2 discusses the related literature. Section 3 presents the model. Section 4 introduces the functional forms, describes the estimation procedure and the data. Section 5 presents the results. Section 6 presents a series of robustness checks. Finally, Section 7 concludes. A separate appendix with additional results and robustness checks is posted online.

2 Related Literature

Our work is related to the large literature on firm valuation.[1] Our approach is most closely related to the supply approach to valuation developed in BXZ, but extended to a setup in which multiple and heterogeneous capital inputs and labor, not just physical capital, can contribute to a firm's market value. Following the important work of Ohlson (1995), a large number of empirical studies have regressed firm valuation ratios on several firm characteristics. We differ from these empirical studies in that the relationship between valuation ratios and firm characteristics in our model has a structural interpretation, in particular, the parameter estimates can be linked to the firm's technology. Thus, our estimates and functional forms can guide future research of models of investment with several capital inputs.

Our paper is also related to the asset pricing literature on intangible capital and firm risk.

[1] See BXZ for an overview of the firm valuation literature in Finance, Economics, and Accounting.

Eisfeldt and Papanikolaou (2013) estimate the value of organization capital using a model of the sharing rule between a firm's owners and its key talent. They show that firms with more organization capital are riskier than firms with less organization capital. Following Lev and Radhakrishnan (2005), the authors construct a broad measure of organization capital using selling, general and administrative (SG&A) expenses. Indeed, this measure includes not just the value of the labor force (as it accounts for the costs of training workers), but also knowledge capital (as it often includes R&D expenditures), and brand capital (as it accounts for advertising expenses), among other non-capital input related expenses.[2] Since our goal is to decompose the value of the firm and to understand the relative contribution of labor and the different intangible capital inputs for firms' market value, we focus on measures of the separate components rather than using this broad measure of organization capital (in the robustness section below we also consider a version of the model using this broad measure of intangible capital). This is important because the type of intangible capital (knowledge versus brand capital) that matters the most for firm value varies significantly across industries. Hansen, Heaton, and Li (2012) study the risk characteristics of intangible capital. Li and Liu (2012) and Vitorino (2014) study the importance of intangible capital in a q-theory model via structural estimation. We build on their work by considering a general model that includes both knowledge and brand capital, and also frictions in the adjustment of the labor input. By considering a more general framework we can provide a more accurate assessment of the contribution of each input to firm value.

A growing literature has further shown the importance of intangible capital for corporate decisions. Falato, Kadyrzhanova, and Sim (2014), building on earlier work by Corrado, Hulten, and Sichel (2009) and Corrado and Hulten (2010), show that intangible capital is the most important firm-level determinant of corporate cash holdings, with the rise in intangible capital being a fundamental driver of the secular trend in U.S. corporate cash holdings over the last decades. We differ from these studies because our structural model allows us to measure the market value of the capital inputs (given by the product of the shadow price of the input with the book-value of the input), not just the book value of the inputs. As we show, a firm-value decomposition based on book value of the capital inputs is significantly different from a decomposition based on the

[2] As discussed in Peters and Taylor (2017), companies typically report SG&A and R&D separately, but Compustat almost always adds R&D expenses to SG&A, reporting them together in the variable XSGA.

market value of the inputs. In addition, the total book value of the inputs is unable to explain the time-series and cross-sectional variation in market values across firms.

Peters and Taylor (2017) propose a new measure of Tobin's Q that accounts for intangible capital, and show that their measure is a superior proxy for explaining total firm investment in physical and intangible capital. Our structural model of the firm, which also incorporates intangible capital, provides a quantitative decomposition of Tobin's Q into the value of each capital input according to the optimal corporate policies including labor hiring, and investment in physical and intangible capital. In addition, Peters and Taylor (2017) document that the investment-q relation works best in high-tech sectors. Andrei, Mann, and Moyen (2018) confirm this finding and show that it can be rationalized in an augmented investment model with corporate learning about firms' cash flows. Consistent with these findings, we show that an augmented investment model with two types of intangible capital and quasi-fixed labor inputs matches the data in the high-tech sector particularly well, further improving the fit relative to the one-physical-capital input model.

An important strand of the asset pricing literature documents the effect of labor-market frictions on stock returns and firm value.[3] The theoretical approach in this paper is related to the work of Merz and Yashiv (2007), who build upon the earlier work by Cochrane (1991). Merz and Yashiv (2007) consider an aggregate representative firm facing adjustment costs in both capital and labor, and focus on the estimation of the production and adjustment costs functions. They show that adding labor adjustment costs substantially improves the model's ability to capture the dynamics of the aggregate stock market value. We build on the Merz and Yashiv (2007)'s setup by including two additional types of costly intangible capital. Further, extending the model to the firm-level allows us to exploit not only time-series data, but also firm-level cross-sectional data. Building on Merz and Yashiv (2007), Belo, Lin, and Bazdresch (2014) shows that labor hiring negatively predicts future returns in the cross section both in model simulations and in the data. In our work, we focus on equity valuation ratios and we provide a structural estimation of the frictions in (physical and intangible) capital and labor.

Our work is also related to the large literature on labor demand and capital investment which

[3]A partial list of studies linking labor market variables to asset prices includes Mayers (1972), Fama and Schwert (1977), Campbell (1996), Jagannathan and Wang (1996), Jagannathan, Kubota, and Takehara (1998), Santos and Veronesi (2005), Boyd, Hu, and Jagannathan (2005), and Lustig and Van Nieuwerburgh (2008). The interpretation of the empirical facts in these studies is silent about the production-side of the economy (technology).

investigates the importance of capital and labor adjustment costs to explain investment and hiring dynamics.[4] The estimated economic magnitude of adjustment costs is still subject to debate. For example, Shapiro (1986) shows that large estimates of labor adjustment costs are important to match investment and hiring dynamics, particularly for non-production workers. Hall (2004), however, shows estimates for both capital and labor adjustment costs that are negligible at the two-digit SIC industry level. We add to this literature by providing structural estimates of adjustment costs for multiples types of capital and labor inputs based on financial market data.

Finally, our paper also contributes to the literature on the importance of capital heterogeneity. Abel (1985) provides closed-form solutions for firm market value in a q-theory model with several factors of production, and Abel and Eberly (2001) provide empirical evidence on the relevance of capital heterogeneity. Using a dataset of Japanese firms, Hayashi and Inoue (1991) find strong empirical support for the relationship between aggregate capital growth and Tobin's Q derived in a model with multiple capital goods. Similarly, Chirinko (1993) estimates an investment model with multiple capital inputs and adjustment technologies, and finds significant evidence in favor of capital heterogeneity. Gonçalves, Xue, and Zhang (2017) show that an investment-based model with both physical capital and current assets can simultaneously capture the cross-sectional variation in average returns across a large set of portfolios. These papers, however, do not look at valuation moments, and hence at the firm value decomposition, as we do here.

3 The Model of the Firm

The model is a neoclassical model of the firm as in LWZ/BXZ (we use their notation whenever possible), extended to a setup with several quasi-fixed inputs. Time is discrete and the horizon infinite. Firms choose costlessly adjustable inputs (e.g., materials, energy) each period, while taking their prices as given, to maximize operating profits (revenues minus the expenditures on these inputs). Because we treat labor and intangible capital as quasi-fixed inputs, the labor costs

[4]See, for example, on capital: Cooper and Haltiwanger (1997), Caballero et al. (1995), Cooper, Haltiwanger, and Power (1999) and Cooper and Haltiwanger (2006); on labor: Hamermesh (1989), Bentolila and Bertola (1990), Davis and Haltiwanger (1992), Caballero and Engel (1993), Caballero, Engel, and Haltiwanger (1997), Cooper, Haltiwanger, and Willis (2015); on joint estimation of capital and labor adjustment costs: Shapiro (1986), Galeotti and Schiantarelli (1991), Hall (2004), Merz and Yashiv (2007) and Bloom (2009). Bond and Van Reenen (2007) survey the literature, and Hamermesh (1996) reviews a set of direct estimates of labor adjustment costs.

and the investments in intangible capital are excluded from our definition of operating profits. Then, taking these operating profits as given, firms optimally choose the physical and intangible capital investments, hiring, and debt to maximize their market value of equity.

To save on notation, we denote a firm's i set of capital and labor input stocks at time t, as $\boldsymbol{K_{it}}$(variables in bold represent a vector). This set includes the physical capital stock (K_{it}^P), the labor stock (L_{it}), the knowledge capital stock (K_{it}^K), and the brand capital stock (K_{it}^B). Similarly, we denote a firm's i set of investments in the inputs (hiring in the case of the labor input) at time t, as $\boldsymbol{I_{it}}$. This set includes the investment in physical capital (I_{it}^P), the investment in labor stock, that is, gross hiring (H_{it}), the investment in knowledge capital (I_{it}^K), and the investment in brand capital (I_{it}^B).

The laws of motion of the firm's capital inputs and labor force are given by:

$$K_{it+1}^P = I_{it}^P + (1-\delta_{it}^P)K_{it}^P \tag{1}$$

$$L_{it+1} = H_{it} + (1-\delta_{it}^L)L_{it} \tag{2}$$

$$K_{it+1}^K = I_{it}^K + (1-\delta_{it}^K)K_{it}^K \tag{3}$$

$$K_{it+1}^B = I_{it}^B + (1-\delta_{it}^B)K_{it}^B, \tag{4}$$

where δ_{it}^P, δ_{it}^K and δ_{it}^B are the exogenous depreciation rates of physical, knowledge and brand capital, respectively. δ_{it}^L is the employee quit rate, i.e., the rate at which the workers leave the firm for voluntary reasons.

3.1 Technology

The operating profit function for firm i at time t is $\Pi_{it} \equiv \Pi(\boldsymbol{K_{it}}, \boldsymbol{X_{it}})$, in which $\boldsymbol{X_{it}}$ denotes a vector of exogenous aggregate and firm-specific shocks. Firms incur adjustment costs when investing and hiring. The adjustment costs function is denoted $C_{it} \equiv C(\boldsymbol{I_{it}}, \boldsymbol{K_{it}})$. This function is increasing and convex in investment and hiring, and decreasing in the capital stocks and the labor force. We specify the functional forms in the empirical section below.

We assume that the firm's operating profit function and adjustment costs function are both homogeneous of degree one. As we show in Section 3.3, these assumptions allow us to obtain a closed-form expression for the firm's equilibrium market value in the model, which depends on the model parameters and on firm-level data. This greatly simplifies the estimation of the model because it allow us to structurally estimate the model directly using real data, as opposed to indirectly through simulated data (for example, using the simulated method of moments).

It is important to note that the assumption of homogeneity of degree one of the operating profit function does not necessarily imply the assumption of perfect competition, nor that the firm's production function exhibits constant returns to scale. To show this more formally, in the online appendix, we consider the maximization problem of a firm with a production function that exhibits decreasing returns to scale in physical capital (for simplicity, we assume that the firm only uses this capital input for production), and that faces a downward sloping demand curve (that is, it has market power, potentially driven by intangible capital). The firm's stock of intangible capital does not enter in the production function directly as an input, but it affects demand: a higher stock of intangible capital increases consumers' willingness to pay for the firm's goods. We show that, in this setup, there exists a set of parameter values in which the operating profit function is homogeneous of degree one in the two capital inputs (physical and intangible capital). This example further shows that knowledge or brand capital might matter for firms' value not necessarily through their effect on production (as in the case of physical capital and labor), but through their effect on consumers' willingness to pay, and hence on demand and profits. Indeed, our model specification with an homogeneous of degree one operating profit function in the four inputs can be reinterpreted as a model of the firm in which the production function has decreasing returns to scale in physical capital and labor and the firm has market power, potentially driven by the firm's stock of intangible (knowledge or brand) capital.

3.2 Taxable Profits and Firm's Payouts

We allow firms to finance investments with debt. At the beginning of time t, firm i issues an amount of debt, denoted B_{it+1}, which must be repaid at the beginning of time $t+1$.[5] r_{it}^{B} denotes the gross corporate bond return on B_{it}.

[5] We include debt in the model to better match the data, but we keep the financing side of the firm simple and frictionless to focus on the production side of the firm as the main driver of the model fit.

We can write taxable corporate profits as operating profits minus intangible capital investments (which are expensed), labor costs, physical capital depreciation, adjustment costs, and interest expense:

$$\Pi_{it} - I_{it}^K - I_{it}^B - W_{it}L_{it} - \delta_{it}^P K_{it}^P - C_{it} - (r_{it}^B - 1)B_{it}.$$

Thus, adjustment costs are expensed, consistent with treating them as foregone operating profits.

Let τ_{it} be the corporate tax rate. The payout of firm i is then given by:[6]

$$D_{it} \equiv (1-\tau_t)[\Pi_{it} - C_{it} - I_{it}^K - I_{it}^B - W_{it}L_{it}] - I_{it}^P + B_{it+1} - r_{it}^B B_{it} + \tau_t \delta_{it}^P K_{it}^P + \tau_t(r_{it}^B - 1)B_{it}, \quad (5)$$

in which $\tau_t \delta_{it}^P K_{it}^P$ is the depreciation tax shield and $\tau_t(r_{it}^B - 1)B_{it}$ is the interest tax shield.

3.3 Equity Value

Firm i takes the stochastic discount factor, denoted M_{t+1}, from period t to $t+1$ as given when maximizing its cum-dividend market value of equity:

$$V_{it} \equiv \max_{\{\mathbf{I}_{it+\triangle t}, \mathbf{K}_{it+\triangle t+1}, B_{it+\triangle t+1}\}_{\triangle t=0}^{\infty}} E_t \left[\sum_{\triangle t=0}^{\infty} M_{t+\triangle t} D_{it+\triangle t} \right], \quad (6)$$

subject to a transversality condition given by $\lim_{T\to\infty} E_t[M_{t+T}B_{it+T+1}] = 0$, and the laws of motion for the capital inputs and labor given by equations (1) to (4).

Let $P_{it} \equiv V_{it} - D_{it}$ be the ex-dividend equity value. In the appendix we show that, given the homogeneity of degree one of the operating profit and adjustment costs functions, the firm's value maximization implies that:

$$P_{it} + B_{it+1} = q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K K_{it+1}^K + q_{it}^B K_{it+1}^B, \quad (7)$$

[6]Note that physical capital investment and intangible capital investments are treated differently given the different accounting rules. Investment in physical capital is spread out over time and expensed as depreciation, while the intangible capital costs (which in our case are R&D and advertising expenses) are mostly treated as expenses at the time that they occur.

in which

$$q_{it}^{P} \equiv 1 + (1 - \tau_t)\partial C_{it}/\partial I_{it}^{P} \tag{8}$$

$$q_{it}^{L} \equiv (1 - \tau_t)\partial C_{it}/\partial H_{it} \tag{9}$$

$$q_{it}^{K} \equiv (1 - \tau_t)\left[1 + \partial C_{it}/\partial I_{it}^{K}\right] \tag{10}$$

$$q_{it}^{B} \equiv (1 - \tau_t)\left[1 + \partial C_{it}/\partial I_{it}^{B}\right], \tag{11}$$

and $\partial C_{it}/\partial x$ denotes the first derivative of the adjustment costs function with respect to variable x, and q_{it}^{P}, q_{it}^{L}, q_{it}^{K} and q_{it}^{B} measure the shadow prices of physical capital, labor, knowledge capital, and brand capital, respectively (the Lagrange multipliers of equations (1) to (4)). The valuation equation (7) is simply an extension of Hayashi (1982)'s result to a multi-factor inputs setting. This equation allows us to compute the firm's market value from real variables only, namely investment rates and capital/labor inputs. Thus, in contrast with standard valuation approaches in the literature (for example, discounted cash flow method), our approach does not require estimation of future cash flows, nor assumptions about unobserved characteristics such as terminal values or stochastic discount factors.

According to equation (7) the firm's market value is given by the sum of the value of the firm's installed labor and capital inputs. This additive feature allows us to compute the fraction of firm value that is attributed to each input (henceforth referred simply as "input-shares") in a straightforward manner as follows:

$$\mu_{it}^{P} = \frac{q_{it}^{P} K_{it+1}^{P}}{q_{it}^{P} K_{it+1}^{P} + q_{it}^{L} L_{it+1} + q_{it}^{K} K_{it+1}^{K} + q_{it}^{B} K_{it+1}^{B}} \tag{12}$$

$$\mu_{it}^{L} = \frac{q_{it}^{L} L_{it+1}}{q_{it}^{P} K_{it+1}^{P} + q_{it}^{L} L_{it+1} + q_{it}^{K} K_{it+1}^{K} + q_{it}^{B} K_{it+1}^{B}} \tag{13}$$

$$\mu_{it}^{K} = \frac{q_{it}^{K} K_{it+1}^{K}}{q_{it}^{P} K_{it+1}^{P} + q_{it}^{L} L_{it+1} + q_{it}^{K} K_{it+1}^{K} + q_{it}^{B} K_{it+1}^{B}} \tag{14}$$

$$\mu_{it}^{B} = \frac{q_{it}^{B} K_{it+1}^{B}}{q_{it}^{P} K_{it+1}^{P} + q_{it}^{L} L_{it+1} + q_{it}^{K} K_{it+1}^{K} + q_{it}^{B} K_{it+1}^{B}}. \tag{15}$$

The fundamental goal of the empirical analysis is to characterize these shares, including their variation across industries and over time.

4 Estimation Methodology

In this section we specify the functional forms and describe the estimation procedure. In addition, we describe the data, including the measurement of the intangible capital stocks, and report descriptive statistics of the key variables used in the analysis.

4.1 Functional Forms

The valuation equation (7) only requires the specification of the adjustment costs function, not of the operating profit function. We consider the following quadratic adjustment costs function:

$$C_{it} = \frac{\theta_P}{2}\left(\frac{I_{it}^P}{K_{it}^P}\right)^2 K_{it}^P + \frac{\theta_L}{2}\left(\frac{H_{it}}{L_{it}}\right)^2 W_{it}L_{it} + \frac{\theta_K}{2}\left(\frac{I_{it}^K}{K_{it}^K}\right)^2 K_{it}^K + \frac{\theta_B}{2}\left(\frac{I_{it}^B}{K_{it}^B}\right)^2 K_{it}^B, \tag{16}$$

in which W_{it} is the wage rate (which the firm takes as given), and $\theta_P, \theta_L, \theta_K, \theta_B > 0$ are the parameters that control the magnitude of the adjustment costs of each input. Labor adjustment costs are proportional to the firm's wage bill, as in Bloom (2009). This helps to make the units of the labor adjustment costs (measured in number of workers) similar to the other capital inputs which are measured in (real) dollar values, an adjustment that is important for the empirical results below.

This functional form implies that the shadow prices of labor and the capital inputs can be inferred from firm-level data on investment, hiring, capital and labor stocks, wages, and taxes, and are given by:

$$q_{it}^P \equiv 1 + (1-\tau_t)\theta_P\left(\frac{I_{it}^P}{K_{it}^P}\right) \tag{17}$$

$$q_{it}^L \equiv (1-\tau_t)\theta_L\left(\frac{H_{it}}{L_{it}}\right)W_{it} \tag{18}$$

$$q_{it}^K \equiv (1-\tau_t)\left[1 + \theta_K\left(\frac{I_{it}^K}{K_{it}^K}\right)\right] \tag{19}$$

$$q_{it}^B \equiv (1-\tau_t)\left[1 + \theta_B\left(\frac{I_{it}^B}{K_{it}^B}\right)\right]. \tag{20}$$

We adopt a simple quadratic adjustment cost specification for parsimonious reasons and to avoid parameter proliferation. There are several implicit assumptions in our simple specification that are worth discussing. First, we assume that adjustment costs depend on the gross (as opposed to net) flow of the inputs. For example, in the case of labor, firms may incur adjustment costs even if the number of workers does not change (net flow is zero) but there is labor turnover, because the firm needs to hire and train the new workers. The importance of using gross labor flows instead of net flows is consistent with the empirical evidence in Hamermesh (1995). For consistency, we adopt the same specification for all the inputs.

Second, we only consider smooth adjustment costs and thus ignore non-convex adjustment costs that lead to lumpy investment. According to the analysis in Section 3.3, the assumption of smooth adjustment costs allow us to derive a closed form expression for the firm's equilibrium value as a function of firm real variables and model parameters, which greatly simplifies the estimation of the model. This specification is reasonable in our context because our sample (described below) consists of publicly listed firms for which the evidence of inaction/lumpiness in investment is more limited than for establishment-level data.

Third, we assume symmetry across positive and negative input adjustments (e.g., in the case of labor, the adjustment cost of hiring or firing one worker is the same), and we also assume that the curvature of the adjustment costs function is two (quadratic). In robustness checks (Section 6.1), we relax the symmetry assumption and we also consider a more flexible representation of the adjustment costs function in which we allow the value of the curvature parameter to be different from two (and different across inputs). In both alternative specifications, we obtain a model fit that is similar to the simpler specification considered here.

4.2 Estimation Procedure

The valuation equation (7) links firm value to the value of its labor and capital inputs. Since firm values are not necessarily stationary, it is useful to scale the variables in this equation for estimation purposes. Accordingly, we scale the variables in the equation by dividing them by the sum of the firm's capital inputs, which we denote as A_{it+1}, a measure of the firm's total (effective) assets given by $A_{it+1} \equiv K^P_{it+1} + K^K_{it+1} + K^B_{it+1}$. For scaling purposes, we do not include labor in this definition of total assets because labor is measured in different units (number of workers as opposed to dollars in real terms). Accordingly, we write a firm's valuation ratio ($VR_{it} \equiv (P_{it} + B_{it+1})/A_{it+1}$) as:

$$VR_{it} = q_{it}^{P}\frac{K_{it+1}^{P}}{A_{it+1}} + q_{it}^{L}\frac{L_{it+1}}{A_{it+1}} + q_{it}^{K}\frac{K_{it+1}^{K}}{A_{it+1}} + q_{it}^{B}\frac{K_{it+1}^{B}}{A_{it+1}}. \tag{21}$$

The left-hand side (LHS) of equation (21) can be directly measured in the data from equity price and debt data (and measures of the capital stocks, which we discuss below). The right hand side (RHS) of equation (21) is the predicted valuation ratio from the model, which we will denote as $\widehat{VR}_{it}$, and depends on firm-level real variables and model parameters.

Equation (21) establishes an exact relationship between a firm's observed valuation ratio and its model-implied valuation ratio at each point in time. Using equation (21) and firm-level data to directly estimate the model parameters is challenging, however. First, firm-level data can be very noisy and measurement error in the data can make estimation at the firm-level very sensitive to outliers. Second, firm-level moments are sensitive to firm entry and exit, and are likely affected by missing observations. These are important considerations in our analysis due to the length of the firm-panel studied and because the R&D and advertising expenses data needed to construct the knowledge capital and brand capital stocks are missing for a nontrivial fraction of the firms in Compustat (as discussed in Section 4.4 below).

To circumvent the previous issues while maximizing the use of the information in our sample, we estimate the model parameters using portfolio-level moments as in BXZ, which in turn follow the original approach in LWZ. The use of portfolio-level moments, a common practice in the asset pricing literature, has several attractive features in our context. First, it allows us to reduce the noise in the firm-level data, and hence obtain arguably more accurate estimates that are less subject to an attenuation bias (akin to a more accurate estimation of the betas in empirical asset pricing). Second, portfolio-level moments are arguably less sensitive, and hence more stable, to firm entry and exit, and to missing firm-level observations. Finally, it allows us to characterize the data in a more parsimonious manner because the number of portfolios is naturally smaller than the number of firms in the data.

We proceed as follows. In theory, at each point in time, any cross-sectional moment of the observed firm-level valuation ratios in the LHS of equation (21) should be equal to any corresponding cross-sectional moment of the model-implied firm-level valuation ratios in the RHS of equation (21). Accordingly, for each portfolio j and for each year t, we compute the cross-sectional mean (XSM) observed valuation ratio (VR_{jt}^{XSM}) and the model-implied valuation ratio ($\widehat{VR}_{jt}^{XSM}$) of the firms in the portfolio. Specifically, we compute:

$$VR_{jt}^{XSM} = \sum_i \frac{VR_{it}}{N_{jt}}$$

$$\widehat{VR}_{jt}^{XSM}(\Theta) = \sum_i \frac{\widehat{VR}_{it}}{N_{jt}}\ ,\ i \in \text{portfolio } j,$$

where Θ represents the vector of structural parameters, i.e., $\Theta = [\theta_P, \theta_L, \theta_K, \theta_B]$, and N_{jt} is the number of firms in portfolio j at time t. We target cross-sectional mean valuation ratios because these moments capture the economic behavior of a typical (average) firm in the economy, which is what the theory model is designed to study.[7]

We then proceed under the standard assumption that the portfolio-level valuation ratio moments are observed with error by the econometrician:

$$VR_{jt}^{XSM} = \widehat{VR}_{jt}^{XSM}(\Theta) + \varepsilon_{jt}, \tag{22}$$

where ε captures measurement error in the portfolio-level moments.[8] Based on equation (22), we then estimate the model parameters by minimizing the squared distance between the portfolio-level observed and model-implied valuation ratio moments at each point in time:

$$\widehat{\Theta} = \arg\min_{\Theta} \frac{1}{TN} \sum_{t=1}^{T} \sum_{j=1}^{N} \left(VR_{jt}^{XSM} - \widehat{VR}_{jt}^{XSM}(\Theta) \right)^2, \tag{23}$$

where T is the number of years in the sample, and N is the number of portfolios. An attractive feature of our estimation approach is that it corresponds to a simple linear ordinary least squares (OLS) estimation of (modified) portfolio-level average valuation ratios on portfolio-level averages

[7] Arguably, our model is less appropriate for the valuation of superstar firms, such as Apple or Facebook, which are likely to derive a large part of their market value from features not captured by our model.

[8] Mismeasured components of the valuation ratio such as the market value of debt and the capital inputs can be better observed by firms than by econometricians. Furthermore, the intrinsic value of equity can temporarily diverge from the market value of equity.

of firm-characteristics. This is due to the linear relationship between the model-implied valuation ratio and the parameters, combined with the use of portfolio-level cross-sectional means as target moments.[9]

Finally, we compute Newey-West standard errors with lag equal to three years, to account for possible cross-sectional and time-series correlations.

We note that our estimation procedure differs from the portfolio-level estimation procedure in BXZ and LWZ in two important ways. First, in each period t, we target the cross-sectional portfolio-level mean instead of targeting a portfolio-level aggregate valuation ratio (which aggregates each portfolio-level characteristic separately using the Fama and French 1993 approach). This modification is important to recover the true firm-level structural parameters since, as we show in the online appendix, the procedure in BXZ/LWZ is subject to an aggregation bias which precludes the parameter estimates from having a structural interpretation (see also, Zhang 2017, Gonçalves, Xue, and Zhang 2017, and Belo, Deng, and Salomao 2019, for a discussion of the aggregation bias in LWZ). Naturally, the ability to recover the firm-level structural parameters is crucial to provide a proper decomposition of the market value of the firm. Second, our estimation procedure requires the model to match the realized time series of the portfolio-level valuation ratios as close as possible, not just their time series average as in BZX and LWZ. This is important in the context of our analysis because the contribution of some of the inputs for firm value changes over time. As a result, the time-series data provides relevant information for the identification of the model parameters.

[9] To show this claim more formally, define the following variables: $\overline{VR}_{jt} = \frac{1}{N_{jt}}\sum_{i\in j}\frac{\left(P_{jt}+B_{jt+1}-K^P_{jt+1}-(1-\tau_t)K^K_{jt+1}-(1-\tau_t)K^B_{jt+1}\right)}{A_{jt+1}}$ (the modified valuation ratio), $\overline{IPA}_{jt} = \frac{1}{N_{jt}}\sum_{i\in j}(1-\tau_t)\frac{I^P_{it}}{K^P_{it}}\frac{K^P_{it+1}}{A_{it+1}}$, $\overline{HLA}_{jt} = \frac{1}{N_{jt}}\sum_{i\in j}(1-\tau_t)\frac{H_{it}}{L_{it}}\frac{W_{it}L_{it+1}}{A_{it+1}}$, $\overline{IKA}_{jt} = \frac{1}{N_{jt}}\sum_{i\in j}(1-\tau_t)\frac{I^K_{it}}{K^K_{it}}\frac{K^K_{it+1}}{A_{it+1}}$, and $\overline{IBA}_{jt} = \frac{1}{N_{jt}}\sum_{i\in j}(1-\tau_t)\frac{I^B_{it}}{K^B_{it}}\frac{K^B_{it+1}}{A_{it+1}}$. We can then write equation (22) as:

$$\overline{VR}_{jt} = \theta_P\overline{IPA}_{jt} + \theta_L\overline{HLA}_{jt} + \theta_K\overline{IKA}_{jt} + \theta_B\overline{IBA}_{jt} + \varepsilon_{jt} \tag{24}$$

which establishes a linear relation between the portfolio-level modified valuation ratio and portfolio-level characteristics. Thus, our objective function in (23) corresponds to a simple linear OLS regression of equation (24).

4.3 Industry Classification and Portfolio Sorts

We estimate the model both in a pooled sample with all firms in the economy (hence assuming an homogeneous adjustment cost technology across firms), and separately for different industries (hence allowing for heterogeneity in the adjustment cost technology across industries). We consider an industry classification that is based on the labor-skill level (we define this variable below). Accordingly, we split the sample in two industries, which we refer to as low- and high-skill industries. To a first approximation these industries correspond to low- and high-tech sectors of the economy.

The classification of industries according to labor skill (relative to other industry classifications available in the literature) is interesting for the purposes of our analysis because there are a priori reasons to expect that the adjustment costs parameters, and hence the importance of capital and labor inputs for firm value, vary in a systematic way across low- and high-skill industries. First, as discussed in Belo et al. (2017) (also, see references therein) previous empirical studies find that it is more costly to replace a high-skill worker than a low-skill worker. This suggests that the labor adjustment costs parameters should be relatively higher in high-skill industries which, all else equal, imply that labor should also represent a higher fraction of firm value in high-skill industries. Second, Belo et al. (2017) also provide evidence that investment in intangible capital inputs such as R&D expenditures is relatively higher in high-skill industries. Taken together, this suggests that the relative importance of the different capital and labor inputs for firm value should vary across industries with different skill levels. Naturally, the estimation can be performed based on any industry classification. Thus, as a robustness check, we also perform the estimation and corresponding firm-value decomposition based on a standard Fama and French industry classification. We discuss the results in Subsection 6.3.

As noted above, the estimation is performed at the portfolio-level. To minimize the influence of the choice of a particular sorting variable on the results, we estimate the model combining several sorting variables. In choosing the sorting variables, we consider firm-level variables that are likely to generate a large spread in the RHS variables of equation (21), in order to span the entire state space which helps the identification of the model parameters. Accordingly, we consider four sets of portfolios sorted based on the following proxies of the lagged market values of each input: the product of the physical capital investment rate and scaled physical capital, the product of the hiring rate times wages and scaled labor force, the product of the knowledge capital investment rate and scaled knowledge capital, and the product of the brand capital investment rate and scaled brand capital.[10] We then follow Fama and French (1993) in constructing the portfolios. Specifically, we sort all stocks in June of each year t into ten portfolios based on the deciles of the sorting variable of each firm for the fiscal year ending in $t-1$. The portfolios are re-balanced at the end of each June. This procedure gives a total of 40 portfolios across all sorts.

4.4 Data

Here we describe the data and, in particular, we explain how we construct the capital stocks.

Sample selection: Our sample consists of U.S publicly traded firms from 1950 to 2016 (our estimation period starts in 1975 but we use data prior to 1975 to construct the initial intangible capital stocks as described below). Our estimation sample starts in 1975 because that is the year in which the Financial Accounting Standards Board (FASB) required firms to disclose and expense all research and development (R&D) expenditures (used in the construction of the knowledge capital stock) during the year in which these expenses were incurred. The firm-level data are from the Center for Research in Security Prices (CRSP)/Compustat Merged (CCM) – Fundamentals Annual database. We limit our analysis to firms incorporated in the US (Compustat fic="USA") that trade on major stock exchanges (NYSE, AMEX, and NASDAQ) (CRSP exchange codes 1, 2, and 3), for which the native currency is US dollars (Compustat curcd="USD"), and that have information on their ordinary common shares traded (CRSP share codes 10 and 11). We exclude firms with primary standard industrial classifications (SIC) between 4900 and 4999 (regulated utilities) and between 6000 and 6999 (financial services). We drop firm-year observations with missing market values, number of employees or physical capital. In the main sample, we only include firms that

[10]For example, according to equation (21), the value of physical capital is given by $q_{it}^{P}\frac{K_{it+1}^{P}}{A_{it+1}}$. Using equation (17), we can write this value as:

$$q_{it}^{P}\frac{K_{it+1}^{P}}{A_{it+1}} = \left(1 + (1-\tau_t)\theta_P\left(\frac{I_{it}^{P}}{K_{it}^{P}}\right)\right)\frac{K_{it+1}^{P}}{A_{it+1}},$$

and hence by sorting on $\left(\frac{I_{it}^{P}}{K_{it}^{P}}\right)\left(\frac{K_{it+1}^{P}}{A_{it+1}}\right)$, this sorting maximizes the variation of the value of the physical capital stock across firms, as captured by the second term inside the main brackets. Accordingly, the four sorting variables are: $\left(\frac{I_{it}^{P}}{K_{it}^{P}}\right)\left(\frac{K_{it+1}^{P}}{A_{it+1}}\right)$, $\left(\frac{H_{it}}{L_{it}}\right)\left(\frac{W_{it}L_{it+1}}{A_{it+1}}\right)$, $\left(\frac{I_{it}^{K}}{K_{it}^{K}}\right)\left(\frac{K_{it+1}^{K}}{A_{it+1}}\right)$, and $\left(\frac{I_{it}^{B}}{K_{it}^{B}}\right)\left(\frac{K_{it+1}^{B}}{A_{it+1}}\right)$. These values proxy for the market values of each input.

report R&D expenses at least once during their lifetime. Further, we drop firm-year observations with missing advertising or R&D data whenever we are not able to fill in the missing data as described below. The final main sample used for the baseline estimation of the model includes annual data from $4,610$ firms for the period from 1975 to 2016, which corresponds to $52,010$ firm-year observations.

Physical capital data: The initial physical capital stock, K_{i0}^P, is given by net property, plant, and equipment (data item PPENT). The capital depreciation rate, δ_{it}^K, is the amount of depreciation (data item DP) divided by the beginning of the period capital stock. We then construct a measure of the firm's capital stock at current prices. Specifically, we construct an investment-price adjusted capital stock that accounts for changes in the real cost of physical capital investment by repricing last period's capital stock using today's price of investment (P_t^P) as $K_{t+1}^P = K_t^P(1-\delta_t)\frac{P_{t+1}^P}{P_{t+1}^P} + I_{t+1}$. Following Zhang (2017) we infer physical capital investment from the the law of motion of capital by inverting the previous law of motion of physical capital equation and solving for investment (accounting for inflation). This procedure guarantees that the investment and physical capital data are consistent with the law of motion for physical capital in the model.[11]

There is no readily available data on a price of investment index for our physical capital stock, which is mostly composed of structures and equipment. The Bureau of Economic Analysis (BEA) provides a price index for a broad investment series that includes investment in structures, equipment, and intellectual property products (which should be excluded in our analysis because it corresponds to intangible capital), and for each of these three items separately.[12] Thus, we first recover the real values for each of the series "structures" and "equipment" by dividing the nominal values of these two series reported by the BEA in the NIPA table 5.3.5 by their corresponding price indices reported by the BEA in the NIPA table 5.3.4. We then calculate a price index for physical capital that includes only structures and equipment (but not intellectual propriety) in the same manner as the BEA constructs price deflators by dividing the nominal-dollar value of a series by its

[11] Several studies (for example, LWZ) measure investment in physical capital, I_{it}^P, as capital expenditures (item CAPX) minus sales of property, plant, and equipment (item SPPE), and set SPPE to zero if missing. As shown in Zhang (2017), this procedure generates investment series that violate the assumed law of motion of physical capital in several observations. The main reason for this fact is that CAPX excludes acquisitions, that is, increases in the firm's capital stock due to the acquisition of other firms.

[12] The price index for the broad investment series is called "Gross Private Domestic Investment: Fixed Investment: Nonresidential (implicit price deflator)" (series id A008RD3Q086SBEA in FRED).

calculated real value. More specifically, we proceed by dividing the sum of the nominal investment in structures and equipment (reported in the NIPA table 5.3.5) by the sum of the real investment in structures and equipment (recovered by us as described above).

Labor data: The labor stock, L_{it}, is the number of employees (Compustat data item EMP). The labor market data on wage rates and labor quit rates is not available at the firm level for most firms (the firm-level wage bill data in Compustat is missing for more than 80% of the firms in our sample). Thus, we measure these variables at the industry level as follows.

To compute the wage rate per worker, W_{it}, we use annual data from the BEA, National Income and Product Accounts (NIPA), Section 6. The industry-level wage rate per worker is given by the ratio of the total compensation of employees (which includes wage and salary accruals and supplements to wages and salaries) to the total number of employees in the industry. We use compensation of employees by industry from Tables 6.2B-D and the number of (full-time and part-time) employees by industry from Tables 6.4B-D. To merge the wage data with our firm-level data from Compustat/CRSP, we created a mapping between the wage data and the Standard Industry Classification (SIC) 1987 and the North American Industry Classification System (NAICS) 2002 codes using the industries description in the BEA tables.

We measure the annual employee quit rate δ^L_{it} using data for 16 major industry groups based on NAICS codes from the Job Openings and Labor Turnover Survey (JOLTS) available from the Bureau of Labor Statistics (BLS). Because this data is only available since 2001, we extend the data backwards as follows. We estimate a time-varying quit rate by regressing, for each major industry group in JOLTS, the industry level quit rates on real GDP growth, unemployment rate, the labor vacancy rate, and a measure of labor-market tightness.[13] The fit of the regression for each industry is quite good, with a median time-series R^2 of 88% across industries. For each industry, we then extend the quit rate back to cover the entire sample prior to 2001. We also use the same procedure to estimate a time-varying aggregate JOLTS quit rate for the industry group "Total Private" (i.e.,

[13]For the real GDP growth we use the series: Real Gross Domestic Product (U.S. Bureau of Economic Analysis, series A191RL1A225NBEA, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/A191RL1A225NBEA, April 30, 2018). As for the unemployment rate we use the series: Civilian Unemployment Rate [UNRATE](U.S. Bureau of Labor Statistics, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/UNRATE, April 30, 2018.). For the labor vacancy rate we use the Help Wanted Index (HWI) referenced in Barnichon (2010) and provided in Regis Barnichon's website. The HWI is provided at the monthly level, in the regressions we use the yearly average. We construct a measure of labor market tightness as the ratio of the vacancy rate and the unemployment rate.

an overall quit rate), and assign this rate to firms that belong to industries not covered in JOLTS or that have a missing industry code. This procedure allows us to have variation in the employee quit rate both in the cross-section and in the time-series.

Knowledge capital data: Following Falato, Kadyrzhanova, and Sim (2014) we construct the firm's stock of knowledge capital from past expenditures data on R&D (Compustat data item XRD) and using the perpetual inventory method as follows:[14]

$$K_{t+1}^{K} = K_{t}^{K}(1-\delta^{K})\frac{P_{t+1}^{K}}{P_{t}^{K}} + I_{t+1}^{K}, \tag{25}$$

where P_t^K is the BEA price index for intellectual property products, R&D, from the Federal Reserve Economic Data (FRED) database.[15] To implement the law of motion in equation (25) we must choose an initial stock and a depreciation rate. Using the perpetual inventory method, we set the initial stock to:

$$K_0^K = \frac{I_0^K}{g^K + \delta^K - \pi^K(1-\delta^K)},$$

in which I_0^K is the firm's investment in knowledge capital in the first year in the sample, and π^K is the average (net) growth rate of the price index for R&D, which is 3.2% in the sample period used for the estimation. We let g^K be industry specific and set it to be equal to the average growth rate of the R&D investments in that industry; in practice, we consider 10 industries classified based on the average labor skill level of the industry (we describe the labor skill data below). As for the knowledge capital depreciation rate, we use the recommended depreciation rates of R&D assets based on the BEA-NSF dataset as calculated by Li (2012a) and reported for each industry on Li (2012a)'s Table 4, column 3. For the companies/industries not reported in Li (2012a) we use a 15% depreciation rate following Peters and Taylor (2017). Once we have the initial capital stock, we iterate forward using the appropriate depreciation rate, R&D expenses, and investment price index. The investment rate on knowledge capital is then given by the ratio of the current period

[14] See also Lev and Radhakrishnan (2005), Eisfeldt and Papanikolaou (2013), and Li and Liu (2012) for similar applications. The Bureau of Economic Analysis uses a similar methodology to construct a stock of Research and Development capital, see Sliker (2007). Corrado et al. (2004) identifies R&D as the largest source of business investment on intangibles.

[15] Specifically, we use the annual series "Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products: Research and Development (chain-type price index), Index 2009=100" (Y006RG3A086NBEA) provided by the BEA.

investment and the beginning of the period corresponding knowledge capital stock I_t^K/K_t^K.

It is important to note that the XRD item in Compustat often includes not just the R&D expenses reported by companies but also the R&D acquired by companies that is deemed to not have alternative future use (data item RDIP). Thus, we remove RDIP from XRD whenever it is included by Compustat, because, as a write-off, it should not be interpreted as an investment.[16]

We treat missing R&D data as follows. In 1972, the APB Opinion No. 22 made the disclosure of R&D expenditures in financial statements mandatory and the SEC started to require the reporting of R&D in the Annual 10-K reports (SEC No.125). However, until 1975, when the FASB started to require the expensing of all R&D expenditures during the year incurred, disclosure of R&D in the firms' financial statements was still limited. Given this, after 1975, we assume that if R&D is missing, it corresponds to zero. Also, in order to use as much data as possible, and because there is a significant share of companies that reports R&D prior to 1975, we construct the stocks of knowledge capital starting in 1970 (whenever possible). Even tough we only estimate the model starting in 1975, this procedure allows us to mitigate the negative impact on our analysis of the potential mis-measurement of the initial knowledge capital stock for the firms present in the sample which have data prior to 1975.

Brand capital data: The construction of the brand capital stock is analogous to the construction of the knowledge capital stock. Following Belo, Lin, and Vitorino (2014) and Vitorino (2014), we construct the firm's stock of the brand capital from past expenditures data on advertising (Compustat data item XAD) and using the perpetual inventory model as follows:[17]

$$K_{t+1}^B = K_t^B(1-\delta^B)\frac{P_{t+1}^B}{P_t^B} + I_{t+1}^B, \tag{26}$$

where P_t^B is the advertising industry's output price index (PPI), available from the Bureau of

[16]RDIP is included in XRD whenever the item XRD_FN (XRD footnote) in Compustat has values BW (Includes In-Process, Acquired, or Purchased Research and Development) or BV (Includes In-Process, Acquired, or Purchased Research and Development and engineering expense or customer- or government-sponsored research and development). Note also that RDIP (In-Process R&D Expense) is coded by Compustat as negative, so to remove RDIP from XRD we subtract the absolute value of RDIP from XRD.

[17]According to Compustat, advertising is usually an indirect operating cost that is reported by companies as a selling expense within SG&A. Whenever advertising expenditures are reported separately from SG&A (in a note or in a supplementary schedule in the 10-K reports), Compustat includes them in data item XAD. **Ptok, Jindal, and Reinartz (2018)** discusses in detail the use of several Compustat variables to operationalize various Marketing-related constructs. Their results suggest that XAD from Compustat is a satisfactory measure of advertising spending.

Labor Statistics.[18] The initial stock of brand capital is set to:

$$K_0^B = \frac{I_0^B}{g^B + \delta^B - \pi^B(1-\delta^B)},$$

in which I_0^B is the firm's investment in brand capital in the first year in the sample, and π^B is the average (net) growth rate of the price index for advertising expenses, which is 5.4% in the sample period used for the estimation. We let g^B be industry specific and set it to be equal to the average growth rate of advertising expenses in that industry (using the same 10-industry classification described in the construction of the knowledge capital stock). As in Vitorino (2014), we use a depreciation rate for brand capital of 20%. Once we have the initial capital stock, we iterate forward using the depreciation rate, advertising expenses, and investment price index. The investment rate on brand capital is then given by the ratio of the current period investment and the beginning of the period corresponding brand capital stock I_t^B/K_t^B.

We treat the missing XAD data as follows. In 1994, the SEC passed Financial Reporting Release 44 (FRR 44), which eliminated the disclosure requirement of advertising expenditures in public firms' annual reports. Before the passage of FAR 44 (which became effective on December 20, 1994), public firms were required to report advertising spending if it exceeded 1 percent of their total sales (according to the SEC Release AS-125, which became effective for financial statements for periods ending on or after Dec 31, 1972). Based on this we calculate the brand capital stocks using data starting in 1972 when companies start to report advertising expenses in the "Supplementary Income Statement Information" schedule. We impute missing advertising data based on the observed Selling, General and Administrative (SG&A) expenses using the firm-level average ratio of advertising expenses-to-SG&A ratio for the years in which neither of these values

[18]Specifically, the price index for brand capital is the average yearly Producer Price Index by Industry: Advertising Agencies (U.S. Bureau of Labor Statistics, "Producer Price Index by Industry: Advertising Agencies" [PCU541810541810], retrieved from FRED, Federal Reserve Bank of St. Louis). Because this data series only starts in 1996, we extrapolate backwards using as predictors "Personal Consumption Expenditures: Chain-type Price Index, Index 2009=100 (BEA)", "Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products (chain-type price index), Index 2009=100", "Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products: Research and Development (chain-type price index), Index 2009=100 (BEA)", "Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products: Entertainment, Literary, and Artistic Originals (chain-type price index), Index 2009=100 (BEA)", and "Private fixed investment, chained price index: Nonresidential: Intellectual property products: Software, Index 2009=100 (BEA)" from the period of 1929 until 1995). (Note: the IPP software series only starts in 1959 so it only enters as a predictor after 1959).

is missing. Given the different disclosure requirements throughout the years, however, we cap the imputed amount at 1% of sales for the years from 1972 through 1993 (to make the imputed values consistent with the reporting standards). We exclude from the sample all the firms with missing XAD during the entire sample period.

Discussion on the limitations of the intangible capital data: Our measurement of the firm-level intangible capital stocks follows previous work whenever possible taking into account legal reporting requirements and how the data is treated in Compustat. In constructing these stocks, however, it is important to recognize that there are several implicit empirical choices that one has to make, and which can potentially have some impact on the results. For example, although the total amount of intangible capital (here, knowledge capital and brand capital) of a firm is given by the sum of externally acquired and internally created intangible capital, our measures of knowledge and brand capital relate to internally created intangible capital only. We focus on internally created intangible capital due to several data limitations. First, when one company purchases another, externally generated or acquired intangible assets are recorded by companies as assets in the balance sheet and are recorded by Compustat in item INTAN (Intangibles) which corresponds to the sum of the items INTANO (Other Intangibles) and GDWL (Goodwill). Other intangibles refers to identifiable intangible assets.[19] Goodwill is recorded as the residual between the cost of an acquired business and the fair market value of net tangible assets and identifiable intangible assets. This means that even though Goodwill is recorded as an intangible asset on the acquiring company's balance sheet, goodwill is often contaminated by non intangibles (such as market premium for physical assets).

In addition, the treatment and reporting of externally created intangible assets has not been consistent over time. For example, up to 2001, when a firm acquired another company, it could choose to recognize all the intangible assets that were acquired at that time ("purchase method") or only those that had been previously recorded by the acquired entity at the time of a previous acquisition ("pooling-of-interests method"). After 2001, however, with accounting rule SFAS 141, the "pooling-of-interests" method was eliminated as an option. This leads to inconsistency in the treatment of intangibles across and within companies. Moreover, the breakdown of INTAN into

[19] An asset is identifiable when it is separable –that is, it is capable of being separated or divided from the acquired entity and sold, transferred, licensed, rented or exchanged (regardless of whether there is an intent to do so)–, or when it arises from contractual or other legal rights. Examples of identifiable intangible assets include computer software, licences, trademarks, patents, films, copyrights and import quotas.

"Other Intangibles" and "Goodwill" is only available in Compustat after 2001 (Accounting rule SFAS 142 issued in 2001 requires greater disclosure of information about goodwill and intangible assets).

Additional firm-level variables: We measure debt, B_{it}, as net total debt. Specifically, we measure net debt as long-term debt (Compustat data item DLTT) plus short-term debt (data item DLC), minus cash (data item CHE), setting them to zero when they are missing. The market value of equity, P_{it}, is the closing price per share (data item PRCC_F) times the number of common shares outstanding (data item CSHO). For firms with different fiscal-year ends the price matches the firm's fiscal year (and thus the timing of the accounting data).

We measure the tax rate, τ_t, as the statutory corporate income tax for the highest bracket from the Commerce Clearing House, annual publications, until 2010, and from Deloitte's corporate tax rates annual publications after 2010. Stock variables with subscript t ($t+1$ for debt) are measured and recorded at the end of year t, while flow variables with subscript t are measured over the course of year t and recorded at the end of year $t+1$.

Labor skill industry classification data: We classify an industry to be a low- or high-skill industry based on the percentage of workers in that industry that work on occupations that require a high level of training and preparation (high-skill workers) using the Specific Vocational Preparation (SVP) index from the Dictionary of Occupational Titles (DOT), available from the Department of Labor, and employee data from the BLS, Occupational Employment Statistics (OES) program. The data is from Belo et al. (2017), available from the authors' webpages. The base industry-level data is available at the three-digit SIC level before and including year 2001, and at the four-digit NAICS level after 2001. An industry is classified as a high labor-skill industry if it belongs to a 3-SIC or 4-NAICS industry in which the percentage of high-skill workers in that industry (defined in Belo et al. 2017 as variable PSKILL) is above the median of the cross-sectional distribution of the PSKILL variable. Conversely, we classify an industry as a low labor-skill industry if the percentage of high-skill workers in that industry is below the median of the cross-sectional distribution of the industry-level PSKILL variable. Because the data only refers to the period from 1991 to 2013, for the period from 1975 to 1990, we use an average of the data from 1991 to 2001 and, for the period from 2014 to 2016, we use an average of the data from 2002 to 2013. The industry classification of each firm is very stable over time.

4.5 Summary Statistics

Panel A in Table 1 reports key summary statistics (time-series average of the cross-sectional median, denoted as "Average", and standard deviation, denoted "S.D.") of the observed valuation ratios and their model-implied components according to equation (21), in the pooled sample where all firms are included ("All Firms" sample), and for the low- and high-skill industries ("Low Skill" and "High Skill" subsamples, respectively). All ratios are winsorized at the top and bottom (if the variable admits negative values) 1% to mitigate the impact of outliers in the analysis.

The average valuation ratio across all firms is 1.95. This valuation ratio is higher in high-skill industries than in low-skill industries, 2.04 versus 1.57, respectively.

[Insert Table 1 here]

In terms of the average size of the scaled (by assets) capital and labor inputs, in the pooled sample, the largest scaled input is labor (using lagged wages as implied by equation (21)), which amounts to 61% of total assets. The second largest input is physical capital, with 42% of total assets. The ratio of the knowledge capital stock to total assets is 38%. The smallest capital stock is brand capital with 10% of total assets. The relative magnitude of the ratios varies across the different labor-skill industries. For example, the scaled physical capital stock is higher in low-skill than in high-skill industries, 63% versus 38% of total assets, respectively. Similarly, the scaled brand capital stock is higher in low-skill than in high-skill industries, 14% versus 9% of total assets, respectively. Conversely, the scaled knowledge capital stock is lower in low-skill than in high-skill industries, 13% versus 44% of total assets, respectively.

The shadow prices of the labor and capital inputs in equations (17) to (20) are determined by the investment/hiring rates. Thus, understanding the properties of the investment/hiring rates is useful for understanding the time-series properties of the value of the inputs. Panel A in Table 1 shows that, in the pooled sample, investment in knowledge capital has the highest average rate (28%), while investment in labor, the gross hiring rate, has the lowest rate (16%). The investment and hiring rates are all higher, and more volatile, in high-skill than in low-skill industries. Panel

B in Table 1 also reports the investment and hiring rate cross-correlations in the low- and high-skill industries. The table shows that, as expected, the investment/hiring rates are all positively correlated among each other. The correlations range between 14% and 51%. These correlations are significantly smaller than one, thus suggesting that there is at least some independent variation in the shadow prices, and hence the market values, of the different capital and labor inputs.

5 Estimation Results

This section reports the main empirical findings.

5.1 Firm-Value Decomposition Based on Book-Values

Before estimating the model, we can make a preliminary assessment of the relative importance of each input for firm value based on the book-value of the inputs. We can then use these book-value input-shares as a benchmark to interpret the market-value input-shares obtained from the estimated model.

If adjustment costs are zero, the shadow prices of the capital and labor inputs in equations (17) to (20) are simply one (physical capital), zero (labor), $(1-\tau_t)$ (knowledge capital), and $(1-\tau_t)$ (brand capital). As a result, in this scenario, the value of each input is basically given by its book-value (adjusting for the tax rate), and the fraction of firm value attributed to each capital input (input-shares) can be computed from equations (12) to (15). In the case of labor, the shadow price is zero because firms cannot sell nor buy workers in the same manner that they buy or sell capital goods. Hence, without positive labor adjustment costs, the book value of labor is zero.

To properly characterize the data, we summarize the properties of the firm-level input-shares in the economy using two different aggregation procedures. In the first procedure, we compute an aggregate-level (or within each industry-skill type) share (denoted "Aggregate"). Specifically, we compute this aggregate measure by separately summing up the numerator and the denominator of equations (12) to (15) across all the firms in the economy (or industry), calculate the aggregate share as the ratio of those aggregated values, and report the time-series average of this share for each input. This aggregate measure provides a straightforward way to interpret the data but puts significantly more weight on the large firms in the economy. Thus, in the second procedure, we compute in each year and for each input, the cross-sectional median input-shares, and report the time series mean of these input-shares (denoted "Average") for each input.[20]

[Insert Table 2 here]

Table 2 reports the firm's book-value decomposition using the two aggregation procedures for the pooled sample, and separately for the low- and high-skill industries. As it turns out, for the book-value decomposition, the aggregate and the average share measures provide a similar characterization of the data, so we focus the discussion here on the aggregate level measure.

As noted, without labor adjustment costs, the value of the installed labor force is zero.[21] In the pooled sample, the most important input is physical capital, which represents about 64.33% of firms' book-value. The second most important input is knowledge capital which represents 22.78% of firms' book-value, and the least important input is brand capital which represents about 12.89% of firms' book-value. These numbers vary significantly across industries. The importance of physical capital and brand capital for the book value of the firm is significantly higher in the low-skill than in the high-skill industries, with 71.66% versus 61.49%, respectively, for physical capital, and with 18.23% versus 10.89%, respectively, for brand capital. Conversely, the value of knowledge capital is significantly lower in low-skill than in high-skill industries, with 10.11% versus 27.62%, respectively. Thus, consistent with the summary statistics of the scaled capital and labor ratios reported in Table 1, Panel A, this decomposition is suggestive that the relative importance of the two intangible capital inputs varies across industries: knowledge capital is more important in high-skill than in low-skill industries, while brand capital is more important in low-skill than in high-skill industries.

Turning to the analysis of the quality of the model, the bottom panel in Table 2 provides three measures of model fit. Specifically, the panel reports: i) the cross-sectional R^2 (denoted $XS-R^2$) of

[20]If we compute directly the cross-sectional median share of each input and report the time-series mean of these input-shares, the sum of the shares does not add to 100% because the medians are not additive. Thus, we proceed as follows. First, in each year, we compute the median scaled value of each input (for example, for physical capital, this corresponds to the cross sectional median of $q_{it}^{P}\frac{K_{it+1}^{P}}{A_{it+1}}$), then we compute the implied median total firm value as the sum of the median value of each input, and finally we compute the corresponding input-shares as the ratio of the median scaled values of each input as a fraction of the total median firm value. We then report the time-series mean of this measure for each input.

[21]We note, however, that most of the R&D expenses used to construct the knowledge capital stock are labor compensation. Specifically, on average, 45% of R&D expenses correspond to salaries of personnel who are engaged in R&D projects. This suggests that the value of the knowledge capital stock also partially captures the value of labor.

the best linear fit of the average portfolio-level valuation ratio plotted against the average portfolio-level predicted valuation ratio; ii) the time-series R^2 (denoted $TS-R^2$) of the pooled portfolio-level data (basically, this measure is the square of the correlation between the predicted and the realized portfolio-level valuation ratios, pooling the data for all portfolios as one large time series); and iii) the average mean absolute error (m.a.e.), computed as the time series mean of the absolute errors of the error term of each portfolio (time series average of $|VR_{jt} - V\hat{R}_{jt}|$), as a fraction of the average absolute value of the valuation ratio of the portfolio (denoted m.a.e./$\overline{\text{VR}}$).

According to the three metrics considered here (and, in particular, given the negative cross-sectional and time-series R^2's), the variation in the book value of the inputs is unable to capture the cross-sectional and time-series variation of the valuation ratios of the portfolios. This result shows that the time variation in the shadow inputs (and hence in the market value of the inputs beyond their book values) is likely to be an important ingredient for the ability of the model to capture the large cross-sectional and time-series variation in the valuation ratios observed in the data.

5.2 Parameter Estimates and Model Fit

Table 3, column (1), reports the point estimates of the adjustment costs parameters for the pooled sample. These estimates are $\theta_P = 1.50$ for physical capital, $\theta_L = 11.26$ for labor, $\theta_K = 12.47$ for knowledge capital, and $\theta_B = 3.24$ for brand capital. The adjustment costs coefficients are statistically significant for labor and knowledge capital, which implies that we cannot reject the hypothesis that these inputs are subject to positive adjustment costs. The positive point estimates of the adjustment costs parameters are consistent with the model which assumes that the input adjustment costs function is increasing in the investment/hiring rates.

[Insert Table 3 here]

Turning to the analysis of the model fit, Table 3 shows that the model performs quite well when estimated in the pooled sample, both in the cross-sectional and in the time-series dimensions. The cross sectional R^2 is high, 94%, even tough the model estimation does not explicitly targets this moment. The time-series R^2 is 61%. These results stand in sharp contrast with the poor fit of the version of the model without adjustment costs and in which case firm value is only driven by variation in the book-value of the inputs. In terms of average valuation ratio errors, the model scaled mean absolute error (m.a.e./$\overline{\text{VR}}$) is quite low, about 22%. Thus, the model is able to explain about 78% of the portfolio-level observed valuation ratios (the remaining 22% reflect, for example, measurement and misspecification errors).

Turning to the analysis of the point estimates of the model for the low- and high-skill industries, Table 3, columns (2) and (3) show that all the adjustment cost parameters are positive and, except for brand capital in the high-skill industries, we can reject the hypothesis that these parameters are zero. The estimate of the adjustment cost parameter for labor increases with the average labor-skill level of the industry, with a value of $\theta_L = 7.66$ in the low-skill industries compared to $\theta_L = 10.64$ in the high-skill industries. Going in the opposite direction, the adjustment costs parameters for physical capital, knowledge capital, and brand capital decrease with the average labor-skill level of the industry.

The model is particularly good at capturing the time-series variation in the valuation ratios in the high-skill industries, with a time-series R^2 of 60%, whereas the time-series fit in the low-skill industries is more modest, where the R^2 is 38%. The cross-sectional fit is quite good in both industries, with a cross-sectional R^2 above 94%. Figure 1 provides a visual description of the good fit of the model in the cross section. This figure shows the scatter plot across portfolios of the time-series average of the cross-sectional median valuation ratio observed in the data against the value predicted by the model. The model mean absolute error in the high-skill industries is low, 22% of the average observed valuation ratio in those industries, and in the low-skill industries it is about 38% of the average observed valuation ratio.

[Insert Figure 1 here]

The better (time series) fit of the model in high-skill industries is consistent with the findings in Peters and Taylor (2017) and Andrei, Mann, and Moyen (2018) who show that, in a one-capital-input model, the observed valuation ratios (market to book ratios) explain physical capital investment better in high-tech sectors than in low-tech sectors. Part of the reason for this pattern can be explained by the significantly higher volatility of the valuation ratios in the high-skill industries (which implies more variation to explain), as reported in Table 1, especially given that this higher volatility does not seem to be driven by random noise but rather by a higher variance in the value of the capital (and labor) inputs (Panel A in Table 1 shows that the characteristics of the firms in the high-skill industries are more volatile).

5.3 Firm Value Decomposition Based on Market-Values

The parameter estimates allow us to compute the model-implied shadow prices of each input, and hence evaluate the contribution of each input for firm value (input-shares) based on each input's market value. The approach used here is analogous to the analysis of the book-value decomposition reported in Subsection 5.1. Specifically, using the estimates reported in Table 3 columns (1) to (3), we compute, for each firm and in each year, the values of $q_{it}^{P}\frac{K_{it+1}^{P}}{A_{it+1}}$, $q_{it}^{L}\frac{L_{it+1}}{A_{it+1}}$, $q_{it}^{K}\frac{K_{it+1}^{K}}{A_{it+1}}$, and $q_{it}^{B}\frac{K_{it+1}^{B}}{A_{it+1}}$, that is, the model-implied scaled value of each capital/labor input. We then substitute these values in equations (12) to (15) to compute, in each year, the share of the firm's value attributed to each capital/labor input, both for the pooled sample, and for the low- and high-skill industries separately.[22]

In Table 4 we summarize the properties of the input-shares in the economy using the two (aggregate and average) measures discussed in Subsection 2. In addition, to investigate the degree of input-share heterogeneity in the firm-level data, Figure 2 shows the box plot (across all years) of the firm-value input-shares for the low- and high-skill industries.

[Insert Table 4 here]

[Insert Figure 2 here]

Table 4, column (1) shows that the four inputs are important determinants of firms' market values. When the model is estimated across all firms, and using the aggregate input-share measure, the share of physical capital is 30.36%, the share of installed labor is 22.53%, the share of knowledge capital is 38.28%, and the share of brand capital is the remaining 8.83%. When we use the average input-share measure, the inferences are similar, but the importance of physical and brand capital is smaller, while the importance of labor and, especially, of knowledge capital, is higher. Specifically,

[22] Note that, with this procedure, the input-shares add up to 100% by construction. This does not mean that the model explains the entire variation of the firm's value without any error. As discussed in Subsection 5.2, based on the m.a.e./$\overline{\text{VR}}$ ratio, the model captures between 69% (low skill) and 78% (high skill) of the firm's valuation ratio. Thus, our analysis here provides a decomposition of the firm value that is explained by the model.

here, the share of physical capital is 21.85%, the share of installed labor is 26.61%, the share of knowledge capital is 46.84%, and the share of brand capital is the remaining 4.70%. This analysis reveals that physical capital accounts for less than 31% of the firm's total market value on average. Overall, this analysis shows clearly that, in the modern economy, the non-physical capital inputs (intangible capital and labor) are important determinants of firms' market values.

Turning to the analysis across labor-skill industries, the results reported in Table 4 columns (2) and (3) show that the relative importance of the capital/labor inputs exhibits substantial variation across these industries. The average fraction of firm value attributed to labor and, especially, to knowledge capital, increases with the average labor-skill level of the industry. In the low-skill industries, the share of labor is on average 14.33% using the aggregate measure (18.14% using the average measure), whereas in the high-skill industries this share is 20.85% using the aggregate measure (24.32% using the average measure). Similarly, in the low-skill industries, the share of knowledge capital is on average 20.34% using the aggregate measure (22.19% using the average measure), whereas in the high-skill industries this share is 43.24% using the aggregate measure (51.36% using the average measure).

Going in the opposite direction, the fraction of firm value attributed to physical capital and to brand capital decreases with the average labor-skill level of the industry. In the low-skill industries, the share of physical capital is on average 40.16% using the aggregate measure (42.64% using the average measure), whereas in the high-skill industries this share drops to 29.91% using the aggregate measure (20.91% using the average measure). Similarly, in the low-skill industries, the share of brand capital is on average 25.17% using the aggregate measure (17.03% using the average measure), whereas in the high-skill industries this share drops to 6.02% using the aggregate measure (3.41% using the average measure).

Interestingly, the pattern of the input-shares across industries reveals that, even tough intangible capital is an important component of the firm's market value across all industries, the type of intangible capital (knowledge or brand capital) that matters the most for firm value varies substantially across industries. In low-skill industries, the two intangible capital inputs have approximately the same importance (with average input-shares between 17% and 25%) for a combined share of around 40%−45%. But, in high skill industries, knowledge capital is significantly more important for firm value than brand capital. Here, the combined share of the two intangible capital inputs is between 50% and 55%, and knowledge capital accounts for about 90% of this combined share. This result highlights the importance of considering heterogeneous measures of intangible capital in empirical work.

Turning to the analysis of the firm-level heterogeneity in the firm-level input-shares, Figure 2 reveals that there is substantial heterogeneity in input-shares both in low- and high-skill industries. For example, for physical capital, the 25th and 75th percentile in low-skill industries are around 22% and 50%, respectively, and in high-skill industries they are around 10% and 30%, respectively. For labor, the 25th and 75th percentile in low-skill industries are around 5% and 25%, respectively, and in high-skill industries they are around 10% and 40%, respectively. For knowledge capital, the 25th and 75th percentile in low-skill industries are around 10% and 30%, respectively, and in high-skill industries they are around 25% and 70%, respectively. Finally, for brand capital, the 25th and 75th percentile in low-skill industries are around 5% and 30%, respectively, but in high skill industries, the mass of the share is concentrated at very low levels, all below 10%. Thus, the relatively low share of brand capital for firm value in high-skill industries is a pervasive feature across all firms in these industries.

5.4 Firm Value Decomposition Over Time

The previous analyses focus on the time-series average of the input-shares in the full sample from 1975 to 2016. Here, we perform the same analysis across different sub-periods to investigate if the relative importance of the different inputs has changed over time (we do not re-estimate the model parameter values because these are assumed to be constant over time).

[Insert Table 5 here]

[Insert Figure 3 here]

Table 5 reports the time series averages of the share of each input across decades: 1970s (1975-1979), 1980s (1980-1989), 1990s (1990-1999), 2000s (2000-2009), and 2010s (2010-2016). To save space, we report only the input-shares computed using the aggregate input-share measure. The results using the average input-share measure are similar, consistent with the analysis in Subsection

5.3. Figure 3 provides a visual description of the trends in the input-shares in the data, both for the low- and the high-skill industries.

The table and the figure allow us to identify interesting patterns. The importance of physical capital for firm value is significantly lower in recent years when compared to the earlier part of the sample, while the importance of intangible capital, broadly defined, is significantly higher. But the type of intangible capital that has gained more importance in recent years varies across industries. In low-skill industries, there is a significant increase in the importance of brand capital, but a relatively small increase in the importance of knowledge capital. In high-skill industries, there is a significant increase in the importance of knowledge capital, but effectively no change in the importance of brand capital. The importance of labor for firm value does not exhibit an obvious trend as its share has remained relatively constant over the sample period.

Taken together, the analysis in this section further highlights the importance of the non-physical capital inputs for firm value, especially in the most recent decades, and in high-skill industries, where the non-physical capital inputs account for, on average, more than 78% of firm value. In addition, the change in the relative importance of each input for firm value highlights the importance of targeting the time series of the valuation ratios in the estimation, as opposed to only targeting the cross-sectional time-series means of the valuation ratios (as in LWZ/BXZ).

5.5 Implied Adjustment Costs

We can also use the parameter estimates to characterize the properties of the adjustment cost function of each input. This allows us to assess whether the model fits the data with economically reasonable parameter values, and also to better understand the relatively high importance of labor and intangible capital inputs for firm value (recall that the positive contribution of labor for firm value depends crucially on to existence of positive labor adjustment costs). In addition, the characterization of the adjustment cost function of each input can be useful to guide future research with models featuring multiple capital inputs.

To properly characterize the properties of the adjustment costs of each input we focus on several measures. The first set of measures are based on the realized adjustment costs of each input (that is, ex-post adjustment cost measures which describe the equilibrium outcome). Specifically, using the functional form specification in equation (16) and the parameter estimates, the realized adjustment costs of each input (denoted as CP, CL, CK, and CB) can be computed as a fraction of firm's total annual sales (denoted as Y) as follows:

$$\frac{CP_{it}}{Y_{it}} = \frac{\frac{\theta_P}{2}\left(\frac{I^P_{it}}{K^P_{it}}\right)^2 K^P_{it}}{Y_{it}} \tag{27}$$

$$\frac{CL_{it}}{Y_{it}} = \frac{\frac{\theta_L}{2}\left(\frac{H_{it}}{N_{it}}\right)^2 W_{it}L_{it}}{Y_{it}} \tag{28}$$

$$\frac{CK_{it}}{Y_{it}} = \frac{\frac{\theta_K}{2}\left(\frac{I^K_{it}}{U^K_{it}}\right)^2 K^K_{it}}{Y_{it}} \tag{29}$$

$$\frac{CB_{it}}{Y_{it}} = \frac{\frac{\theta_B}{2}\left(\frac{I^B_{it}}{U^B_{it}}\right)^2 K^B_{it}}{Y_{it}}. \tag{30}$$

The bottom panel in Table 4, columns (1) to (3), reports the average realized adjustment costs of each input, computed as the time-series average of cross-sectional medians of the ratios (27)–(30). To evaluate the degree of firm-level heterogeneity in the realized adjustment costs of each input in the data, Figure 4 shows the box plot of the ratios in the low- and high-skill industries.

Reporting the time-series average of the realized adjustment costs provides a simple way of describing the economic importance of the adjustment costs of each input but, given the convexity of the adjustment costs function, the average of the realized values overstates the actual magnitude of the adjustment costs perceived by firms when making their investment and hiring decisions. Thus, we complement the previous measures with an analysis of the estimated adjustment costs function (the relevant object for firms when making their investment decisions, that is, an ex-ante adjustment costs measure). Specifically, in Panel A of Figure 5, we plot the estimated adjustment cost function of each input (considering an investment/hiring rate from -20% to +20%), and holding fixed the median input-to-sales ratio in the industry (to properly scale the function). To further help in the economic interpretation of the magnitudes of these functions, Panel B of Figure 5 reports the estimated adjustment costs of each input using equations (27) to (30) evaluated at their (average) median investment/hiring rates (using the values reported in the descriptive statistics in Table 1).

[Insert Figures 4 and 5 here]

In the pooled sample, the magnitude of the realized knowledge capital and labor adjustment costs is large, whereas the magnitude of the physical and brand capital adjustment costs is very small. The bottom panel in Table 4, column (1), shows that, on average, the fraction of (annual) sales that is lost due to labor adjustment costs is 6.46% while the corresponding fraction for knowledge capital is 10.05%. The fraction of sales that is lost due to physical capital adjustment costs is only 0.90%, and for brand capital this fraction is 0.49%. Although there is no consensus in the literature on the magnitude of labor and capital adjustment costs, the estimated values of the adjustment costs for these two inputs are within the empirical estimates surveyed in Hamermesh and Pfann (1996), and discussed in Merz and Yashiv (2007). For brand capital, the estimated value of adjustment costs is lower than those estimated in Vitorino (2014) (on average, about 8% of firm's annual sales). The difference is that we are estimating firm-level parameters whereas Vitorino (2014) estimates portfolio-level parameters. In addition, we consider a model with four inputs whereas Vitorino (2014) only considers physical capital and brand capital.

Turning to the analysis of the variation in the size of the adjustment costs across industries, the bottom panel in Table 4, columns (2) to (3), shows that the estimated labor and knowledge capital adjustment costs increase significantly with the average labor-skill level of the industry. The fraction of (annual) sales lost due to labor adjustment costs is on average 2.61% in the low-skill industries, and 6.77% in the high-skill industries. This result is consistent with prior evidence (discussed in the related literature section) that high-skill workers are more costly to replace than low-skill workers, and which motivated our industry classification based on the level of labor skill. Similarly, the fraction of (annual) sales lost due to knowledge capital adjustment costs is on average 2.35% in the low-skill industries, and 13.28% in the high-skill industries.

The positive relationship between the size of the adjustment costs and the average labor-skill of the industry is reversed for brand capital inputs, but the size of the adjustment costs of the two inputs is quite small in both industries (between 0.30% and 1.69% of annual sales), consistent with the analysis across all firms reported in column (1). The box plot of the realized adjustment costs in each industry shown in Figure 4 reveals that there is substantial variation in the realized input adjustment costs across firms. As expected, given the strong link between input-shares and adjustment costs, the pattern in the box-plots of the realized firm-level realized adjustment costs across industries and inputs seems to mimic the pattern and the large variation in firm-level shares of each input reported in Figure 2.

Figure 5 plots the estimated adjustment cost functions of each input for the low- and high-skill industries. These plots confirm that labor and knowledge capital are (ex-ante) relatively more costly to adjust in high-skill industries than in low-skill industries, while the opposite pattern is observed for physical capital and brand capital, consistent with the analysis of the realized adjustment costs. Again, the magnitude of the adjustment costs appears to be reasonable across the plausible range of investment/hiring rates considered here. When the function is evaluated at the (average) median investment/hiring rate of each input, the fraction of annual sales lost due to adjustment costs ranges from 1.06% (physical capital) to 2.16% (labor) in low-skill industries. In high-skill industries, the costs of adjustment are also very low for physical capital and brand capital (1.40% and 0.35% of annual sales, respectively, when evaluated at the (average) median) but are higher for labor and, especially, knowledge capital (5.54% and 11.61% of annual sales, respectively, when evaluated at the (average) median). Thus, while the overall cost of adjusting intangible capital in low-skill industries is small, this cost is high in high-skill industries. Further, the high cost of adjusting intangible capital in high-skill industries is mostly driven by the large cost of adjusting knowledge capital as opposed to by the cost of adjusting brand capital.

Taken together, the parameter estimates reveal that knowledge capital and labor are the two inputs that are most costly to adjust. This finding helps understand the high share of these inputs in the model-implied firm-value decomposition, especially in high-skill industries.

5.6 Model Comparison

To help understand the fit of the model and the relative importance of the various capital inputs for firm valuation, Table 3, columns (4) to (13), reports the parameter estimates and model fit across several restricted versions of the model where we use various subsets of the four inputs. Table 4, columns (4) to (13), reports the corresponding model-implied firm value decomposition and adjustment cost estimates. To save space, for each of the alternative specifications, we only report the results for the low- and high-skill industries (that is, we do not discuss here the results for the pooled sample). Also, we focus most of the discussion on the comparison of the model fit (cross-sectional and time-series R^2) across specifications. To provide a meaningful comparison of the model fit in terms of R^2, we use the same set of firms in the estimation of all models (that is, the sample is the same as the sample used for the estimation of the baseline model), and the observed valuation ratio of each firm (the variable we want to explain) is the same across models, that is, it is scaled by the same sum of the capital inputs (A_t) (so that the variation in the variable we want to explain stays the same across models).

The standard one-physical-capital input model is a natural benchmark for our model. Table 3, columns (4) and (5), show that, consistent with BXZ, this model does a reasonable job explaining the cross-sectional variation in the average valuation ratio across portfolios with a cross-sectional R^2 of 50% in low-skill industries, and of 75% in high-skill industries (versus 95% and 94% in the baseline model). But the one-physical-capital input model fails to explain the time-series variation in the valuation ratios. The time-series R^2 of the one-physical-capital input model is effectively 0% in low-skill industries, versus 38% in the baseline model, and 21% in high-skill industries, versus 60% in the baseline model. Thus, we conclude that the benefit of incorporating additional quasi-fixed inputs in the neoclassical investment model comes primarily from improving the model's ability to capture the time-series variation in the valuation ratios. In addition, this result highlights the importance of examining the time series fit of the model to assess its performance, not just its cross sectional fit (as in LWZ/BXZ).

In addition, the comparison of the estimated realized adjustment costs across model specifications reveals that, when some inputs are ignored, the model estimates provide an improper characterization of the size of an input adjustment costs. As reported in the bottom panels in Table 3, the one-physical-capital input model seems to significantly overestimate the magnitude of physical capital adjustment costs. In the one-physical-capital input model the fraction of annual sales lost due to physical capital adjustment costs is on average between 6.6% and 20.2% of firms' annual sales across industries, versus only 1.2% and 1.5% of firms' annual sales in the baseline model. This result suggests that the one-physical-capital input model attributes to physical capital all the costs of adjusting the other inputs.

Comparing across all model specifications, Table 3 shows that the contribution of each input for the improvement of the model fit varies across industries. For tractability, we focus our discussion here on the time-series R^2 (as opposed to the cross-sectional R^2), because this metric is the most informative for this analysis due to its large variation across model specifications.

Adding labor and, especially, knowledge capital, to the one-physical-capital input model has a first order and similar impact on the quality of the model fit in high-skill industries, whereas adding brand capital has a significantly impact on the quality of the model fit in low-skill industries only. When quasi-fixed labor is added to the one-physical-capital input model, the time-series R^2 in low-skill industries increases from 0% to 14% (compare columns 4 and 6) while it increases from 21% to 39% (compare columns 5 and 7) in high-skill industries. The impact of knowledge capital is even stronger. The time-series R^2 in low-skill industries increases from 0% to 21% (compare columns 4 and 8), and increases from 21% to 50% (compare columns 5 and 9) in high-skill industries, when knowledge capital is added to the one-physical-capital input model. Finally, when brand capital is added to the one-physical-capital input model, the time-series R^2 in low-skill industries increases significantly from 0% to 20% (compare columns 4 and 10), while in high-skill industries it increases very little from 21% to 25% (compare columns 5 and 11).

5.7 Implications for Understanding Value and Growth

A large literature in asset pricing studies the value premium - the difference in average returns between stocks with high valuation ratios (i.e., growth stocks) and stocks with low valuation ratios (i.e., value stocks). Many papers offer potential explanations for the value premium based on the impact of frictions in capital and labor inputs. However, each existing paper focuses mainly on one or, at best, two frictions. For instance, Kogan (2001), Zhang (2005), and Gala 2014 focus on the impact of frictions in physical capital such as costly reversibility to account for the observed value premium. Belo, Lin, and Bazdresch (2014) and Belo et al. (2017) mainly focus on the asset pricing implications of physical capital and labor market frictions. Following Berk, Green, and Naik (1999), Ai and Kiku (2013) model value firms as being assets-in-place intensive, and growth firms as more dependent on growth options, and Ai et al. (2018) generally model investment options as intangible capital. Hansen, Heaton, and Li (2012) also focus on the risk characteristics of intangible capital to account for the value spread in returns, and Eisfeldt and Papanikolaou (2013) emphasize the relevance of organizational capital for asset pricing. This paper complements the existing papers in the literature by providing a comprehensive framework to evaluate quantitatively the relative importance of all of these frictions taken together.

To shed new light on which type of inputs (and corresponding frictions) contribute the most to the market value of growth versus value firms, and thus, to the value premium, we proceed as follows. In each year, we split the firms into value, neutral, and growth firms based on the terciles of the cross-sectional lagged valuation ratio distribution (we compute the terciles separately for low- and high-skill industries). We then use the estimated adjustment costs parameters reported in Table 3, columns (1) to (3), to calculate the time-series average input-shares and adjustment costs for value, neutral, and growth firms.

[Insert Table 6 here]

Table 6 top panel reports the average input-shares across value, neutral, and growth firms. Using the pooled sample estimates, columns (1) to (3) show that, while labor is more important for growth firms than for value firms (share of 29.02 versus 18.98%, respectively), physical capital is more important for value firms than for growth firms (share of 34.87% versus 24.31%, respectively). Regarding the intangible capital, the share of knowledge capital is uniformly higher than the share of brand capital across both value and growth firms. But, in relative terms, growth firms derive relatively more value from knowledge capital that growth firms (40.36% versus 36.44%, respectively) while value firms derive relatively more value from brand capital than from knowledge capital (9.71% versus 6.31%). The analysis using the industry-level parameter estimates reported in columns (4) to (6) (low-skill), and (7) to (9) (high-skill), produces similar patterns when the interpretation is adjusted for the fact that brand capital is overall relatively more important in low-skill industries and knowledge-capital is relatively more important in high-skill industries. The bottom panels in Table 6 display the realized adjustment costs for each input as shares of annual sales. The adjustment costs reflect the patterns we see for input-shares, because, all else equal, higher input adjustment costs imply higher input value.

Taken together, our results are consistent with the idea that value firms derive more value from current assets in the form of physical capital and established brand capital, while growth firms derive more value from growth opportunities in the form of labor and knowledge capital. Frictions in labor, knowledge capital, and brand capital, in addition to frictions in physical capital, seem to be important for understanding the value premium in financial markets.

6 Robustness

To check the robustness of our main findings and, in particular, the importance of non-physical capital inputs for firm value, we re-estimate the model for different data samples and across several perturbations of the empirical procedures. First, we estimate the model assuming a more general adjustment costs function that allows adjustment costs to be asymmetric. Second, we estimate the model using a larger number of portfolios than in the baseline estimation. Third, we consider an alternative industry classification and a different sorting variable for the portfolios. Fourth, we estimate the model directly using firm-level data (as opposed to performing the estimation using portfolios). Fifth, we re-estimate a restricted version of the model without knowledge capital using the sub-sample of firms that were excluded from the main sample due to missing (or always zero) R&D expenses data. Finally, we summarize the results from additional robustness checks (which includes tests using an alternative measure of intangible capital such as organization capital, following Eisfeldt and Papanikolaou 2013) reported in the online appendix.

6.1 Asymmetric Adjustment Costs

In the baseline model, we specify the adjustment costs function to symmetric for parsimonious reasons and to avoid parameter proliferation. This assumption might be at odds with some results in the large investment and labor demand literature, however. For example, Abel and Eberly (1994) and Abel and Eberly (1996) show that allowing for asymmetry in physical capital adjustment costs (e.g., due to investment irreversibility) improves the ability of an otherwise standard neoclassical investment model to explain investment dynamics. Thus, here we consider a more flexible adjustment costs function where we allow the costs of adjusting each input to be asymmetric:

$$\begin{aligned} C_{it} = {} & \tfrac{\theta_P}{v_P^2}\left[\exp\left(-v_P \tfrac{I_{it}^P}{K_{it}^P}\right) + v_P \tfrac{I_{it}^P}{K_{it}^P} - 1\right] K_{it}^P + \tfrac{\theta_L}{v_L^2}\left[\exp\left(-v_L \tfrac{H_{it}}{L_{it}}\right) + v_L \tfrac{H_{it}}{L_{it}} - 1\right] W_{it} L_{it} \\ & + \tfrac{\theta_K}{v_K^2}\left[\exp\left(-v_P \tfrac{I_{it}^K}{K_{it}^K}\right) + v_P \tfrac{I_{it}^K}{K_{it}^K} - 1\right] K_{it}^K + \tfrac{\theta_B}{v_B^2}\left[\exp\left(-v_B \tfrac{I_{it}^B}{K_{it}^B}\right) + v_B \tfrac{I_{it}^B}{K_{it}^B} - 1\right] K_{it}^B. \end{aligned} \tag{31}$$

This function is smooth and homogeneous of degree one, hence it satisfies the requirements for the firm value decomposition result in Subsection 3.3. To help its interpretation, Figure 6 plots this function for the one-capital input case. The parameter θ_i is similar to the single parameter in the baseline specification and controls the size of the adjustment costs of input i. The novel parameter here is v_i which controls the degree of asymmetry of the function. When $v_i > 0$ it is more costly to disinvest (partial irreversibility) than it is to invest. When $v_i < 0$ it is more costly to invest than it is to desinvest. When $v_i \to 0$, this function converges to our standard quadratic adjustment cost specification.[23] Thus, by estimating the parameter v_i, we allow the data to uncover the importance of asymmetry in adjustment costs for our results. Note that, due to the way in which we calculate the investment in the intangible capital inputs, the gross investment rates of these inputs are never negative. Thus, even though the asymmetry parameters for the intangible capital inputs can be estimated, they should be interpreted with caution because the identification of the functional form of the adjustment costs of these inputs is only based on the positive side of investment. Hence, in what follows, we focus most of our discussion on the asymmetry parameters v for the physical capital and labor inputs.

[Insert Figure 6 here]

[Insert Table 7 here]

Table 7 reports the parameter estimates and fit of the model with asymmetric adjustment costs.[24] The evidence of asymmetry for physical capital is not strong in our sample. In low-skill industries, the asymmetry parameter is positive, $v_K = 0.21$, consistent with some irreversibility of investment, but in high-skill industries the parameter is negative, $v_K = -0.25$. In both industries, however, we cannot reject the hypothesis that this asymmetry parameter is zero, that is, that the capital adjustment cost function is symmetric, as in the baseline specification. For labor, there is evidence of some degree of irreversibility in high-skill industries with $v_L = 2.16$ (and this value is more than 4.3 standard errors from zero), but not in low-labor-skill industries with $v_L = 1.19$ (but we cannot reject the hypothesis that this parameter is zero). Thus, in high-skill industries, it is more costly to decrease the labor force (i.e., fire workers) than it is to increase it.

[23]Using l'Hopital's rule, $\lim_{v \to 0} \frac{\theta}{v^2} \left[\exp\left(-v \frac{I}{K}\right) + v \frac{I}{K} - 1 \right] K = \frac{\theta}{2} \left(\frac{I}{K}\right)^2 K$.

[24]Note that the estimation using this adjustment cost specification can no longer be performed using linear OLS. Here, minimizing the objective function in equation (23) requires non linear least squares (NLLS) estimation. We compute bootstrapped standard errors that are robust to cross-sectional and time-series correlation using 20% of the sample with replacement. As shown by Cameron and Miller (2010) bootstrapping controls for the fact that errors can be correlated across portfolios and within portfolios over time.

Turning to the analysis of the impact on model fit, Table 7 shows that, by using the asymmetric adjustment costs function specification, the time-series R^2 of the model increases by 2 percentage points relative to the baseline quadratic adjustment cost specification, from 38% to 40%. The improvement in high-skill industries is slightly higher. In high-skill industries, using the asymmetric adjustment costs function specification, the time-series R^2 of the model increases by 6 percentage points relative to the baseline quadratic adjustment cost specification, from 60% to 66%. This improved fit comes mostly from the asymmetry in the labor adjustment costs discussed above.

Taken together, allowing for asymmetry in the adjustment costs function seems to have only a small impact on the quality of the model fit in our sample, especially in low-skill industries.[25]

6.2 Number of Portfolios

In the baseline estimation, we use 40 portfolios (10 portfolios for each of the 4 portfolio sorts) as test assets. Here, we consider 80 portfolios (20 portfolios for each portfolio sort) as test assets, and investigate the impact on the results.

[Insert Table 9 here]

Table 9, columns (1) to (3), reports the estimation results using this larger number of portfolios as test assets. The point estimates appear to be very similar in magnitude to the point estimates in the baseline estimation. As a result, the model fit and model-implied firm-value decomposition are all quite similar to those obtained in the baseline estimation of the model. This analysis suggests that the point estimates in the baseline estimation are robust to a reasonable variation in the number of portfolios used in the estimation.

6.3 Alternative Portfolio Sorts and Industry Classification

In the baseline analysis we estimate the model using portfolios sorted on proxies of the firm's lagged value of each input. In addition, we split the samples into low- and high-skill industries according to the average share of high-skilled workers in each industry. Naturally, the model can be estimated using other portfolio sorts, and also using other industry classifications.

[25] In the online appendix we report the full set of estimation results for the asymmetric adjustment costs specification, including the model-implied input-shares and adjustment costs. Overall, the results are very similar to those obtained for the baseline adjustment costs specification.

To check the robustness of our main findings to both the portfolio sorting variable and the industry classification, here we report the estimation results using two alternative procedures. In the first procedure, we estimate the model parameters using 15-industry portfolios following the 17-industry Fama and French industry classification (we exclude two industries due to data availability), instead of sorting the portfolios on proxies for the firm's lagged value of each input.[26] The results from this analysis allow us to check the robustness of the findings to the portfolio sorting variable(s). Implicit in this analysis is the assumption that the adjustment cost technology is similar across these industries (we estimate only one set of parameters for all firms). Thus, we also consider a second alternative procedure in which we estimate the model parameters using the same sorting variables of the baseline estimation but perform the estimation separately within each Fama and French industry. The results from this analysis allow us to check the robustness of the findings to the industry classification. To save space, given the large set of results obtained using this second procedure, we discuss here a brief summary of the main results and report the complete analysis using this procedure in the online appendix. Further, we report only the input-shares computed using the aggregate input-share measure.

Table 9, column (4) reports the estimation results using the 15-industry portfolios. The point estimates are similar to those obtained in the baseline estimation. The only noticeable differences are the slope coefficient on brand capital that is larger than in the baseline case ($\theta_B = 11.09$ here versus $\theta_B = 3.24$ in the baseline estimation), and the slope coefficient on labor that is smaller than in the baseline case ($\theta_L = 6.98$ here versus $\theta_L = 11.26$ in the baseline estimation). As a result, the estimated share of brand capital for firm value is slightly higher here than in the baseline model ($\mu_B = 16.87$ here versus $\mu_B = 8.83$ in the baseline estimation), while the estimated share of labor capital for firm value is slightly lower here than in the baseline model ($\mu_L = 12.39$ here versus $\mu_L = 8.83$ in the baseline estimation). More important, the results confirm the importance of the non-physical capital inputs for firm value. Similar to the baseline estimation, the non-physical

[26]We use the 17-industry classification posted on Kenneth French's website. We exclude the industries 14–Utilities and 16–Financial firms due to data availability and sample restrictions. We are left with the following fifteen industries: 1–Food, 2–Mines (Mining and Minerals), 3–Oil (Oil and Petroleum Products), 4–Clths (Textiles, Apparel & Footware), 5–Durbl (Consumer Durables), 6–Chems (Chemicals), 7–Cnsum (Drugs, Soap, Perfumes, Tobacco), 8–Cnstr (Construction and Construction Materials), 9–Steel (Steel Works, etc.), 10–FabPr (Fabricated Products), 11–Machn (Machinery and Business Equipment), 12–Cars (Automobiles), 13–Trans (Transportation), 15–Rtail (Retail Stores), 17–Other.

capital inputs account for roughly 70% of the firm's market value.

The estimation of the model for the different Fama and French industries provides further support for the importance of the non-physical capital inputs for firm value. In the online appendix, we show that although the estimates of the adjustment costs parameters vary across industries, the importance of the non-physical capital inputs is pervasive. The average share of the non-physical inputs ranges from a minimum of 19% in the industry classified as "other", to a maximum of 72% in the high-tech industry. In addition, the analysis of the input-shares in each industry and over time, confirms that the decline in the share of physical capital and the corresponding increase in the share of knowledge capital, also observed in the baseline estimation, is also pervasive across the Fama and French industries. Thus, the decline in the physical-capital share and the increase in the knowledge capital share is not driven by changes in the industry composition in the U.S. economy, but rather seems to be a trend in the overall economy.

6.4 Firm-level Estimation

We perform the baseline estimation using portfolio-level moments. Alternatively, we can estimate equation (24) by ordinary least squares directly on the firm-level data. The advantage of this latter approach is that it does not require us to take a stand regarding a particular sorting variable to create the portfolios. The disadvantage is that, as discussed in Subsection 4.2, this approach is more sensitive to measurement error and the firm-level data is very noisy. Hence, the estimates are likely to be subject to an attenuation bias.

Table 9, columns (5) to (7), reports the estimation results using firm-level data. As expected, the parameter estimates differ somewhat from the baseline estimation. The main noticeable differences are the lower estimates of the labor adjustment cost parameter both in the pooled sample and across the different labor-skill industries, and the higher estimates of the brand capital adjustment cost parameter. This suggests that the measurement error in the labor input may be more severe than the measurement error in the other inputs. As a result, the estimated share of labor for firm value is lower here than in the baseline model, and the estimated share of brand capital for firm value is higher here than in the baseline model.

More important, the estimation results using directly the firm-level data confirm the importance of non-physical capital for firm value. Similar to the baseline estimation, the non-physical capital inputs account for a substantial fraction of firm value, approximately 60% both in the pooled sample and for the different labor-skill industries.

6.5 Alternative Samples

As discussed in Section 4.4, in the main sample, we drop firms that never report (or always report zero) R&D expenses. Ignoring these firms may not be efficient for the purposes of our analysis, however, because these firms may be informative about the importance of the non-physical capital inputs (labor and brand capital) for firm value. Thus, here we estimate a (restricted) version of the model with physical capital, labor, and brand capital only, using the sample of firms that were excluded from the main sample due to missing (or always zero) R&D expenses data. This alternative sample includes $6,541$ firms, and $60,316$ firm-year observations.

Table 9, columns (8) to (10), reports the estimation results obtained using this alternative sample of non-R&D firms. The model fit is even better than the baseline sample/model, especially in the low-skill industries. The times-series R^2 ranges from 55% (low-skill industries) to 66% (high-skill industries), whereas in the baseline sample it ranges from 38% (low-skill industries) to 60% (high-skill industries), respectively.

Across industries, the share of labor ranges from 34.2% (low-skill industries) to 14.52% (high-skill industries), whereas the share of brand capital ranges from 8.30% (low-skill industries) to 22.6% (high-skill industries). Thus, the contribution of labor for firm value is decreasing with labor skill, in contrast with the increasing pattern in the baseline estimation using the main sample. This result suggests that the technology of the R&D versus non-R&D firms may be quite different in terms of the type of workers used. The share of physical capital for non-R&D firms is significantly higher than in the baseline model and ranges from 57.50% (low-skill industries) to 62.87% (high-skill industries). This higher share relative to the baseline sample is perhaps not surprising given that, by definition, the non-R&D firms have zero knowledge capital, which (across most specifications) is the non-physical capital input that contributes the most for firm value in the baseline sample. In addition, the firms that do not perform R&D are likely to be firms from the "old economy", and naturally rely less on innovation and other intangibles, and more on installed physical capital.

Taken together, the average contribution of the non-physical capital inputs for firm value in this alternative sample is still more than 38% of firms' market value across industries. Although this share is smaller than in the baseline model, it is still substantial, thus providing additional support for the importance of the non-physical capital inputs for firm value.

6.6 Additional Robustness Checks

In the online appendix we report the results from additional robustness checks which we briefly summarize here.

First, we consider an adjustment costs specification with a more flexible curvature. Specifically, we estimate the curvature of the adjustment costs function as an additional parameter, instead of imposing a quadratic form. This specification is motivated by the analysis in BXZ who provide evidence in support of a curvature parameter greater than two for the physical capital adjustment cost function. In the online appendix, we show that allowing for a flexible curvature parameter has a very small effect on the model fit and on the conclusions from the model. This is because the estimated curvature parameters do not differ significantly from two. This conclusion differs from BXZ because our estimation method recovers the firm-level structural parameters whereas the procedure in BXZ is subject to an aggregation bias, as discussed in Subsection 4.2.

Second, we consider an alternative (broader) measure of intangible capital, known as organization capital, which is constructed based on Selling, General and Administrative (SG&A) expenses data, following Eisfeldt and Papanikolaou (2013). The main advantage of this measure is that SG&A data is reported for most firms in Compustat, leading to less concerns with missing data. The main disadvantage of this measure is that, as discussed in the Related Literature section, this measure does not allow us to disentangle the importance of the different intangible capital items for firm value because it captures several different types of intangibles. In addition, SG&A includes expenses with training of employees (an investment in labor capital). As a result, this measure is also likely to capture (at least part of) the contribution of labor for firm value.

In the online appendix, we show that although this SG&A measure of intangible capital does not allow us to differentiate across the two types of intangibles (brand and knowledge), the decomposition across physical capital, labor and intangible capital is quite similar to that of the baseline model. In the pooled sample, physical capital accounts for 32.12% of firm value while labor accounts for 16.83%, and organizational capital for 51.05%. For firms in the low-skill industries, labor accounts for 9.99% of firm's market value while in high-skill industries this number rises to 15.40%. While organization capital accounts for a larger share of firm's value in high-skill industries (52% versus 43%), physical capital accounts for more value in low-skill industries (46.88% versus 32.65%).

Finally, we also consider an augmented version of the model which includes organization capital in addition to the four inputs (physical capital, labor, knowledge capital, and brand capital) of the baseline model. To avoid double counting, we remove from the construction of organization capital stock and investment the expenses in R&D and advertising. Again, we find that the overall fit of this augmented model is similar to the fit of the more parsimonious baseline model.

Taken together, the robustness checks analyses show that the importance of the non-physical capital inputs for firm value appears to be a finding that is robust to reasonable variations in the empirical procedures.

7 Conclusion

We incorporate quasi-fixed labor, knowledge capital, and brand capital into the neoclassical model of investment, and estimate the contribution of each input for explaining firm market values in the U.S. economy from 1975 to 2016. The model performs well in explaining both cross-sectional and time-series variation in firms' market values across industries, with a time-series R^2 of up to 61%, and a cross-sectional R^2 of up to 95%. We find that the importance of the non-physical inputs for firm value varies across industries and is substantial, ranging from 70% to 80. On average, while physical capital accounts for 30% to 40% of firms' market value across industries, installed labor force accounts for 14% to 22%, knowledge capital accounts for 20% to 43%, and brand capital for 6% to 25%. We show that financial markets assign large and positive values to the installed stocks of the different types of inputs because they are costly to adjust, especially labor and knowledge capital, thus allowing firms to extract some rents as compensation for the cost of adjusting the inputs. Overall, our analysis provides direct empirical evidence supporting models with multiple capital inputs as main sources of firm value, and shows the importance of the non-physical capital inputs for firm value.

We also document that the contribution of each input for firm value varies over time. The importance of physical capital has decreased substantially over the last four decades, while the importance of knowledge capital input has increased significantly. This trend is pervasive across different industries and thus is not driven by changes in the industry composition in the U.S. economy, but rather reveals a pattern in the overall economy.

Methodologically, our estimation procedure targets portfolio-level cross-sectional moments that allow us to estimate firm-level structural parameters and avoid the aggregation bias of the BXZ/LWZ estimation procedure. This is useful for practical applications because it allows us to compute market values at the firm-level, as opposed to at the portfolio-level, which is naturally more useful in practice. Possible uses of our approach include the valuation of private firms or initial public offerings, guidance in merger and acquisition transactions, among other applications that require estimates of firm values. Moreover, given that our estimation procedure recovers structural adjustment cost parameters, our estimates and functional forms can guide future research with models featuring multiple capital inputs.

Finally, our descriptive quantitative analysis uncovers new questions for future research. For example, what makes brand capital relatively more important in low-skill industries than in high-skill industries, whereas the oppositive pattern holds for knowledge capital? What is the economic source of the large magnitude of adjustment costs in knowledge capital and, to a lesser extent, in labor? The answer to these and other questions may help us understand better the valuation of companies in financial markets.

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Table 1: Descriptive Statistics

Panel A reports the time-series average of the cross-sectional median, and the standard-deviation of selected characteristics of the firm level data across all firms in the economy, and across the low- and high-skill industries. Panel B reports the cross-correlations of the investment/hiring rates in each industry. VR_{it} is the firm's valuation ratio, I_{it}^P/K_{it}^P is the investment rate in physical capital, H_{it}/L_{it} is the investment rate in labor stock (gross hiring rate), I_{it}^K/K_t^K is the investment rate in knowledge capital, and I_{it}^B/K_t^B is the investment rate in brand capital. We also present the descriptive statistics for the stock variables of each input (physical capital, labor, knowledge capital and brand capital) relative to the sum of the three capital inputs (A_{it}, total assets as defined in Section 4.2) and relative to annual sales (Y_{it}). The sample consists of firm-level annual data from 1975 to 2016.

Panel A: Averages and Standard Deviations

	Average			S.D.		
	All Firms	Low Skill	High Skill	All Firms	Low Skill	High Skill
				Valuation ratios		
VR_{it}	1.95	1.57	2.04	4.16	2.83	4.36
				Scaled capital and labor ratios		
K_{it}^P/A_{it}	0.42	0.63	0.38	0.26	0.24	0.25
$(W_{it-1}L_{it})/A_{it}$	0.61	0.54	0.63	1.28	1.62	1.20
K_{it}^K/A_{it}	0.38	0.13	0.44	0.27	0.17	0.26
K_{it}^B/A_{it}	0.10	0.14	0.09	0.16	0.19	0.14
				Investment/hiring rates		
I_{it}^P/K_{it}^P	0.23	0.15	0.26	0.73	0.48	0.77
H_{it}/L_{it}	0.16	0.15	0.17	0.27	0.24	0.28
I_{it}^K/K_{it}^K	0.28	0.21	0.30	0.24	0.18	0.24
I_{it}^B/K_{it}^B	0.25	0.24	0.26	0.24	0.19	0.25
				Capital and labor relative to sales		
K_{it}^P/Y_{it}	0.20	0.25	0.19	0.37	0.31	0.38
$(W_{it-1}L_{it})/Y_{it}$	0.34	0.25	0.36	0.32	0.20	0.34
K_{it}^K/Y_{it}	0.17	0.05	0.21	3.20	0.76	3.48
K_{it}^B/Y_{it}	0.05	0.06	0.05	0.21	0.20	0.21

Panel B: Correlations

	Low Skill			High Skill		
	H_{it}/L_{it}	I_{it}^K/K_{it}^K	I_{it}^B/K_{it}^B	H_{it}/L_{it}	I_{it}^K/K_{it}^K	I_{it}^B/K_{it}^B
I_{it}^P/K_{it}^P	0.49	0.29	0.35	0.51	0.41	0.36
H_{it}/L_{it}		0.14	0.24		0.29	0.32
I_{it}^K/K_{it}^K			0.35			0.47

Table 2: Firm-Value Decomposition Based on Book Values

This table reports the time-series average of the fraction of firm value (input-shares μ) that is attributed to each input based on its book value. This decomposition is done by setting all the adjustment costs to zero. Shares are computed at the aggregate- and average-level according to the procedure described in Subsection 5.1. $XS - R^2$ is the cross-sectional R^2, $TS - R^2$ is the time-series R^2, and $m.a.e./\overline{VR}$ is the mean absolute valuation error scaled by the absolute value of the ratio. The results are reported for the sample of all firms, and also for the sub-samples of low-, and high-skill industries. The sample consists of firm-level annual data from 1975 to 2016.

	All Firms (1)	Low Skill (2)	High Skill (3)
		Aggregate (in %)	
$\bar{\mu}^P$: Physical	64.33	71.66	61.49
$\bar{\mu}^L$: Labor	0.00	0.00	0.00
$\bar{\mu}^K$: Knowledge	22.78	10.11	27.62
$\bar{\mu}^B$: Brand	12.89	18.23	10.89
		Average (in %)	
$\bar{\mu}^P$: Physical	55.97	76.97	51.64
$\bar{\mu}^L$: Labor	0.00	0.00	0.00
$\bar{\mu}^K$: Knowledge	35.69	10.88	40.58
$\bar{\mu}^B$: Brand	8.34	12.15	7.78
		Model fit	
$XS - R^2$	-7.21	-7.63	-7.27
$TS - R^2$	-2.51	-1.78	-2.57
$m.a.e./\overline{VR}$	0.76	0.69	0.77

Table 3: Parameter Estimates and Model Fit

This table reports the parameter estimates and measures of fit for the baseline model specification (columns 1 to 3) and across restricted alternative model specifications with a sub-set of the capital/labor inputs (columns 4 to 13). The estimation uses 40 portfolios sorted based on proxies of the lagged values of the inputs (10 portfolios for each input). θ_P, θ_L, θ_K and θ_B are, respectively, the physical capital, labor, knowledge capital, and brand capital adjustment cost parameters. s.e. stands for Newey-West standard errors with three lags. $XS-R^2$ is the cross-sectional R^2, $TS-R^2$ is the time-series R^2, and $m.a.e./\overline{VR}$ is the mean absolute valuation error scaled by the absolute value of the ratio. The results are reported for the sample of all firms (baseline model only), and also for the sub-samples of low-, and high-skill industries (all specifications). The sample is annual data from 1975 to 2016.

	Baseline			K^P		L		K^P+K^K		K^P+K^B		K^P+L+K^K	
	All Firms	Low Skill	High Skill	Low Skill	High Skill	Low Skill	High Skill	Low Skill	High Skill	Low Skill	High Skill	Low Skill	High Skill
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Parameter estimates													
θ_P	1.50	3.77	2.18	20.55	29.44	10.67	12.79	12.31	13.31	13.59	24.83	5.09	2.68
s.e.	[1.00]	[0.85]	[0.97]	[0.53]	[0.62]	[0.96]	[1.13]	[0.70]	[0.88]	[0.71]	[1.07]	[0.89]	[0.90]
θ_L	11.26	7.66	10.64			10.96	14.54					9.04	10.97
s.e.	[0.69]	[0.84]	[0.66]			[0.97]	[1.02]					[0.84]	[0.67]
θ_K	12.47	18.80	12.28					28.82	14.71			25.14	12.63
s.e.	[0.77]	[1.62]	[0.74]					[2.06]	[0.82]			[1.59]	[0.72]
θ_B	3.24	9.99	2.05							18.92	15.95		
s.e.	[2.05]	[1.46]	[2.48]							[1.78]	[3.14]		
Model fit													
$XS-R^2$	0.94	0.95	0.94	0.5	0.75	0.57	0.74	0.73	0.88	0.64	0.81	0.86	0.92
$TS-R^2$	0.61	0.38	0.60	-0.01	0.21	0.14	0.39	0.21	0.5	0.20	0.25	0.31	0.6
$m.a.e./\overline{VR}$	0.22	0.31	0.22	0.39	0.32	0.36	0.28	0.34	0.25	0.35	0.32	0.32	0.22

Table 4: Firm-Value Decomposition and Adjustment Costs

This table reports the model-implied input-shares (μ) and estimated adjustment costs (CX/Y) for the baseline model specification (columns 1 to 3) and across restricted alternative model specifications with a sub-set of the capital/labor inputs (columns 4 to 13), using the parameters estimates reported in Table 3 to calculate the model-implied input-shares and adjustment costs. Shares are computed at the aggregate- and average-level according to the procedure described in Subsection 5.1. CX/Y is the ratio (in percent) of the implied input adjustment costs-to-sales ratio, computed as the times series average of the cross sectional median of this value. The results are reported for the sample of all firms (baseline model only), and also for the sub-samples of low-, and high-skill industries (all specifications). The sample consists of firm-level annual data from 1975 to 2016.

	Baseline			K^P		$K^P + L$		$K^P + K^K$		$K^P + K^B$		$K^P + L + K^K$	
	All Firms	Low Skill	High Skill	Low Skill	High Skill	Low Skill	High Skill	Low Skill	High Skill	Low Skill	High Skill	Low Skill	High Skill
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
	Firm-value decomposition - Aggregate (in %)												
$\bar{\mu}^P$: Physical	30.36	40.16	29.91	100.00	100.00	73.62	67.83	67.72	55.48	61.03	82.86	50.31	32.5
$\bar{\mu}^L$: Labor	22.53	14.33	20.85			26.38	32.17					19.64	22.08
$\bar{\mu}^K$: Knowledge	38.28	20.34	43.23					32.28	44.52			30.05	45.41
$\bar{\mu}^B$: Brand	8.83	25.17	6.02							38.97	17.14		
	Firm-value decomposition - Average (in %)												
$\bar{\mu}^P$: Physical	21.85	42.64	20.91	100.00	100.00	68.37	52.94	67.59	44.75	71.52	85.82	48.06	22.19
$\bar{\mu}^L$: Labor	26.61	18.14	24.32			31.63	47.06					22.18	25.43
$\bar{\mu}^K$: Knowledge	46.84	22.19	51.36					32.41	55.25			29.76	52.37
$\bar{\mu}^B$: Brand	4.70	17.03	3.41							28.48	14.18		
	Realized adjustment costs (in % of annual sales)												
CP/Y : Physical	0.90	1.22	1.50	6.64	20.23	3.45	8.79	3.98	9.14	4.39	17.06	1.65	1.84
CL/Y : Labor	6.46	2.61	6.77			3.74	9.25					3.09	6.98
CK/Y : Knowledge	10.05	2.35	13.28					3.60	15.91			3.14	13.66
CB/Y : Brand	0.49	1.69	0.30							3.19	2.37		

Table 5: Firm-Value Decomposition Across Decades

This table shows the average aggregate input-shares (μ) (obtained with the aggregation procedure described in Subsection 5.1) across different decades, and using the parameter estimates reported in columns (1) to (3) in Table 3. The results are reported for the sample of all firms, and also for the sub-samples of low-, and high-skill industries. The sample consists of firm-level annual data from 1975 to 2016.

	1970s	1980s	1990s	2000s	2010s
			All firms (in %)		
$\bar{\mu}^P$: Physical	43.15	37.99	28.11	23.93	22.65
$\bar{\mu}^L$: Labor	23.07	18.73	23.01	24.35	24.53
$\bar{\mu}^K$: Knowledge	24.90	33.53	39.89	43.53	44.72
$\bar{\mu}^B$: Brand	8.88	9.75	8.99	8.19	8.10
			Low skill (in %)		
$\bar{\mu}^P$: Physical	48.03	47.15	37.41	34.88	35.89
$\bar{\mu}^L$: Labor	14.83	11.17	14.41	16.67	15.22
$\bar{\mu}^K$: Knowledge	17.40	18.80	19.26	23.04	21.97
$\bar{\mu}^B$: Brand	19.74	22.88	28.92	25.42	26.92
			High skill (in %)		
$\bar{\mu}^P$: Physical	43.87	37.22	27.57	23.65	21.71
$\bar{\mu}^L$: Labor	21.17	17.68	21.21	22.29	22.73
$\bar{\mu}^K$: Knowledge	28.35	38.03	45.43	48.64	50.34
$\bar{\mu}^B$: Brand	6.60	7.06	5.79	5.43	5.22

Table 6: Firm-Value Decomposition Across Value and Growth Firms

This table shows the model-implied aggregate input-shares (μ) (obtained with the aggregation procedure described in Subsection 5.1) and estimated adjustment costs (CX/Y) for value, neutral and growth firms, classified according to the terciles of the cross-sectional lagged valuation ratio distribution. We use the parameters reported in columns (1) to (3) in Table 3 to calculate the model-implied input-shares and adjustment costs. CX/Y is the ratio (in percent) of the implied input adjustment costs-to-sales ratio, computed as the times series average of the cross sectional median of this value. The results are reported for the sample of all firms, and also for the sub-samples of low-, and high-skill industries. The sample consists of firm-level annual data from 1975 to 2016.

	All Firms			Low Skill			High Skill		
	Value	Neutral	Growth	Value	Neutral	Growth	Value	Neutral	Growth
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Firm value decomposition (in %)								
$\bar{\mu}^P$: Physical	34.87	29.74	24.31	43.74	38.41	37.19	33.99	28.84	25.09
$\bar{\mu}^L$: Labor	18.98	22.30	29.02	12.39	14.20	18.90	17.08	21.22	26.65
$\bar{\mu}^K$: Knowledge	36.44	38.75	40.36	19.08	19.66	23.66	42.24	43.87	43.81
$\bar{\mu}^B$: Brand	9.71	9.21	6.31	24.8	27.73	20.25	6.69	6.06	4.45
	Realized adjustment costs (in %)								
CP/Y : Physical	0.46	0.79	1.98	0.71	1.17	2.20	0.74	1.33	3.29
CL/Y : Labor	4.08	5.43	10.85	2.01	2.41	3.93	4.30	5.81	11.41
CK/Y : Knowledge	7.53	8.39	15.08	1.88	2.49	2.75	10.27	11.89	19.13
CB/Y : Brand	0.38	0.46	0.62	1.67	1.79	1.66	0.22	0.29	0.42

Table 7: Parameter Estimates and Model Fit with an Asymmetric Adjustment Cost Specification

This table reports the parameter estimates and measures of fit for the model with adjustment costs function that allows for asymmetric costs. The estimation uses 40 portfolios sorted based on proxies of the lagged values of the inputs (10 portfolios for each input). θ_P, θ_L, θ_K and θ_B are, respectively, the physical capital, labor, knowledge capital, and brand capital adjustment cost parameters. ν_P, ν_L, ν_K and ν_B are, respectively, the physical capital, labor, knowledge capital and brand capital asymmetry adjustment cost parameters. To interpret the asymmetry parameter note that when $v > 0$ its more costly to disinvest than to invest (to capture irreversibility), and vice versa when $v < 0$. s.e. stands for bootstrapped standard errors. $XS - R^2$ is the cross-sectional R^2, $TS - R^2$ is the time-series R^2, and $m.a.e./\overline{VR}$ is the mean absolute valuation error scaled by the absolute value of the ratio. The results are reported for the sample of all firms, and also for the sub-samples of low-, and high-skill industries. The sample consists of firm-level annual data from 1975 to 2016.

	All Firms (1)	Low Skill (2)	High Skill (3)
	Parameter estimates		
Slope			
θ_P	2.33	4.45	3.02
s.e.	[1.32]	[2.31]	[1.42]
θ_L	15.21	9.32	13.41
s.e.	[1.54]	[2.95]	[1.28]
θ_K	18.19	30.29	16.94
s.e.	[1.87]	[6.58]	[1.70]
θ_B	1.42	29.17	0.45
s.e.	[2.94]	[5.86]	[2.14]
Asymmetry			
ν_P	-0.37	0.21	-0.25
s.e.	[0.28]	[0.79]	[0.28]
ν_L	2.55	1.19	2.16
s.e.	[0.56]	[1.29]	[0.50]
ν_K	1.73	2.31	1.47
s.e.	[0.57]	[1.49]	[0.51]
ν_B	-3.57	9.32	-4.96
s.e.	[2.49]	[2.15]	[2.00]
	Model fit		
$XS - R^2$	0.94	0.90	0.94
$TS - R^2$	0.67	0.40	0.66
$m.a.e./\overline{VR}$	0.20	0.31	0.20

Table 8: Firm Value Decomposition and Adjustment Costs for the Asymmetric Adjustment Costs Specification

This table reports the model-implied input-shares (μ) and estimated adjustment costs (CX/Y) for the specification of the model with an adjustment costs function that allows for asymmetric costs, using the parameters estimates reported in Table 7 to calculate the model-implied input-shares and adjustment costs. Shares are computed at the aggregate-level according to the procedure described in Subsection 5.1. CX/Y is the ratio (in percent) of the implied input adjustment costs-to-sales ratio, computed as the time series average of the cross sectional median of this value. The results are reported for the sample of all firms, and also the sub-samples of low-, and high-skill industries. The sample consists of firm-level annual data from 1975 to 2016.

	All Firms	Low Skill	High Skill
	(1)	(2)	(3)
	Firm-value decomposition (in %)		
$\bar{\mu}^P$: Physical capital	31.87	38.48	31.76
$\bar{\mu}^L$: Labor	20.97	13.81	19.04
$\bar{\mu}^K$: Knowledge capital	39.28	22.29	43.93
$\bar{\mu}^B$: Brand capital	7.88	25.42	5.27
	Realized adjustment costs (in %)		
CP/Y : Physical capital	1.46	1.44	2.13
CL/Y : Labor	7.58	2.98	7.58
CK/Y : Knowledge capital	12.41	3.21	15.74
CB/Y : Brand capital	0.31	2.56	0.11

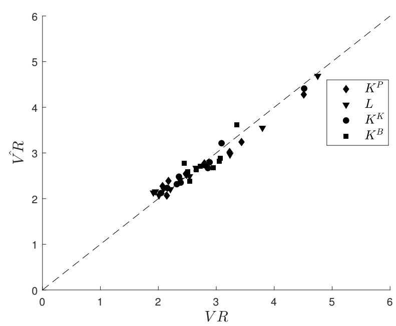
Table 9: Alternative Test Assets, Estimation Method, and Samples

This table reports the parameter estimates, measures of fit, and model-implied input-shares (μ) across alternative empirical procedures. θ_P, θ_L, θ_K, and θ_B are, respectively, the physical capital, labor, knowledge capital, and brand capital slope adjustment cost parameters. s.e. stands for Newey-West standard errors with three lags. $XS-R^2$ is the cross-sectional R^2, $TS-R^2$ is the time-series R^2, and $m.a.e./\overline{VR}$ is the mean absolute valuation error scaled by the absolute value of the ratio. The firm value decomposition reports the input-shares at the aggregate-level according to the procedure described in Subsection 5.1. Columns (1) to (3) report the estimation results using 80 (instead of 40) portfolios sorted based on proxies of the lagged values of the inputs (80 portfolios). Columns (4) reports the estimation results using 17 Fama-French industries as the portfolios (17 Ind.). Columns (5) to (7) report the estimation results by performing the estimation at the firm- (not portfolio-) level (Firm-Level). Columns (8) to (10) reports the benchmark estimation in the sample of firms with missing (or always zero) R&D expense data, using a model with physical capital, labor, and brand capital as the inputs (Non R&D Firms). The results are reported for the sample of all firms, and also for the sub-samples of low-, and high-skill industries (except for the specification 17 Ind.). The sample consists of firm-level annual data from 1975 to 2016.

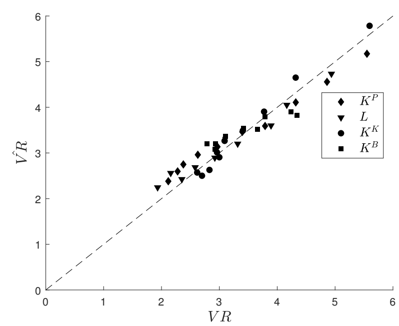
	80 Portfolios			17 Ind.	Firm-Level			Non-R&D Firms		
	All Firms	Low Skill	High Skill	All Firms	All Firms	Low Skill	High Skill	All Firms	Low Skill	High Skill
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					Parameter estimates					
θ_P	2.22	3.13	2.85	2.66	3.72	1.82	3.96	5.65	6.05	7.00
s.e.	[0.73]	[0.48]	[0.71]	[0.75]	[0.37]	[0.52]	[0.42]	[0.99]	[1.56]	[0.88]
θ_L	9.95	7.03	9.20	6.98	5.30	5.15	5.26	7.39	6.38	6.11
s.e.	[0.55]	[0.52]	[0.52]	[1.03]	[0.27]	[0.55]	[0.29]	[0.48]	[0.73]	[0.51]
θ_K	12.92	20.04	12.74	15.16	7.60	18.79	7.33			
s.e.	[0.61]	[1.19]	[0.58]	[1.85]	[1.57]	[2.12]	[1.55]			
θ_B	4.40	11.28	3.97	11.09	13.22	8.16	14.22	5.97	1.09	23.53
s.e.	[1.47]	[1.05]	[1.74]	[3.33]	[1.18]	[1.43]	[1.42]	[2.47]	[2.35]	[2.76]
					Model fit					
$XS-R^2$	0.94	0.91	0.93	0.65	-	-	-	0.94	0.92	0.89
$TS-R^2$	0.58	0.32	0.56	0.34	0.15	0.19	0.14	0.72	0.55	0.66
$m.a.e./\overline{VR}$	0.24	0.35	0.24	0.30	0.64	0.60	0.64	0.25	0.30	0.27
					Firm-value decomposition (in %)					
$\bar{\mu}^P$: Physical	31.88	38.29	31.21	30.98	38.77	40.27	37.42	56.65	57.50	62.87
$\bar{\mu}^L$: Labor	19.55	13.08	17.62	12.39	11.45	11.33	11.33	30.48	34.20	14.52
$\bar{\mu}^K$: Knowledge	38.56	21.32	43.45	39.75	27.79	23.05	32.10			
$\bar{\mu}^B$: Brand	10.02	27.31	7.72	16.87	21.98	25.36	19.15	12.87	8.30	22.61

Figure 1: Cross Sectional Model Fit

This figure plots the time-series average of the model-implied and realized cross sectional average valuation ratios of each portfolio, using the parameter estimates reported in Table 3 columns (2) (low-skill industries) and (3) (high-skill industries), to calculate the model-implied valuation ratios. The sample consists of firm-level annual data from 1975 to 2016.



Panel A: Low Skill



Panel B: High Skill

Figure 2: Distribution of Input Market Value Shares

This figure shows the distribution (box plot) of the estimated firm-level input input-shares (μ) in high- and low-skill industries, using the parameter estimates reported in Table 3, columns (2) and (3), to obtain the input-shares. In each box, the central mark is the median, the edges of the box are the 25th and 75th percentiles. The whiskers extend to the most extreme data points the algorithm considers not to be outliers.

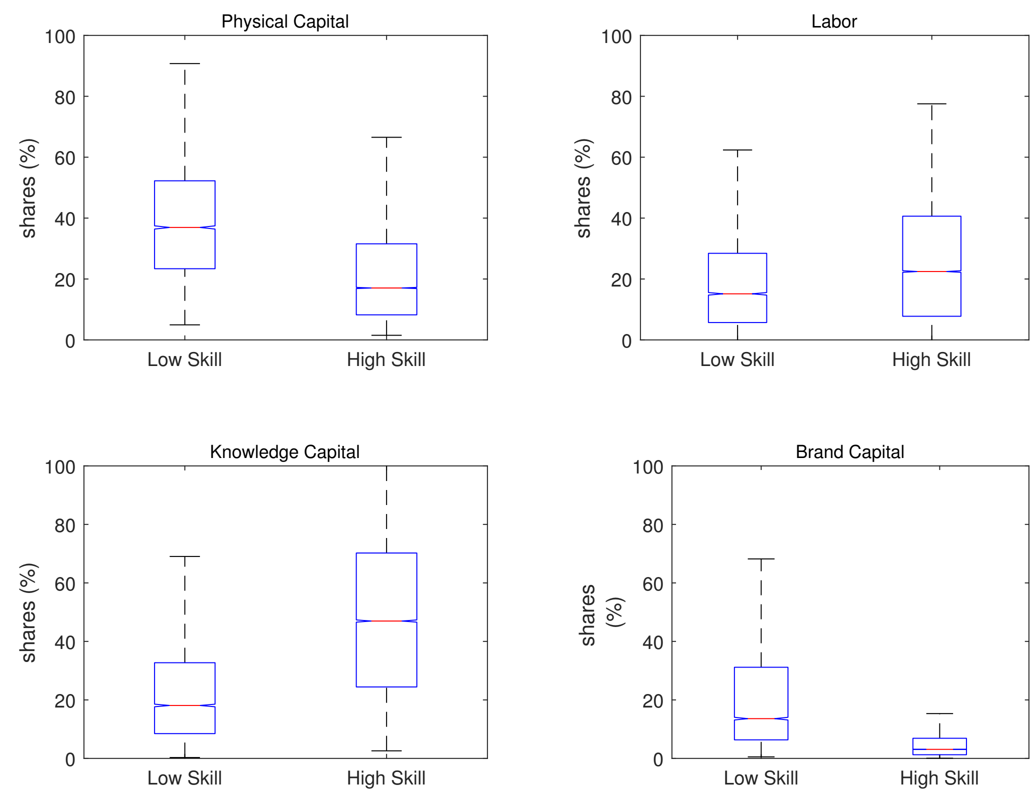
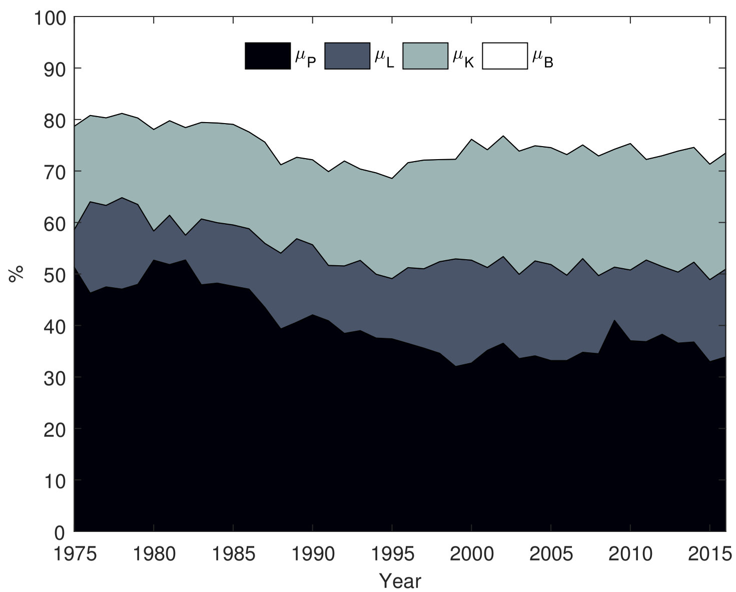
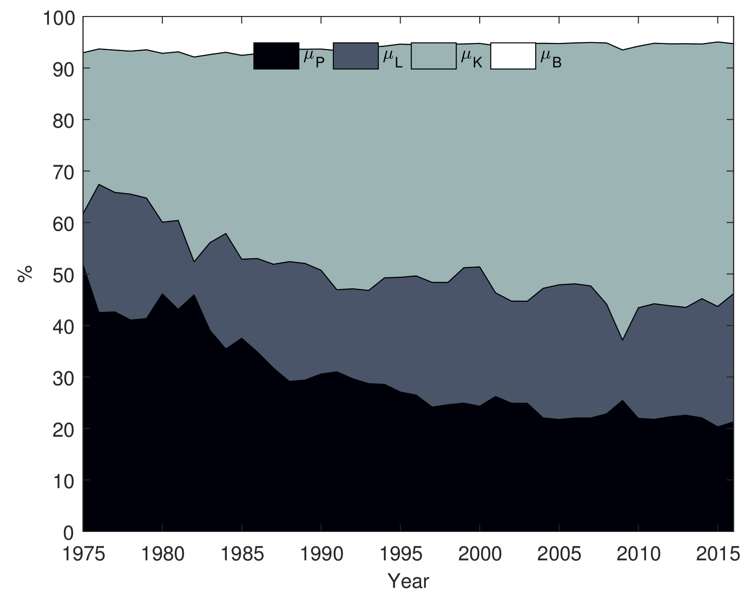


Figure 3: Firm Value Decomposition Over Time

This figure plots the time series of the contribution of each input for the firm's market value (input-shares) in low-skill industries (Panel A) and in high-skill industries (Panel B) implied by the parameter estimates reported in Table 3, columns (2) and (3), and using the aggregate share measure. μ_P is the share of physical capital, μ_L is the share of labor, μ_K is the share of knowledge capital and μ_B is the share of brand capital. Panel A shows the results for low-skill industries, Panel B shows the results for high-skill industries. The sample consists of firm-level annual data from 1975 to 2016.



Panel A: Low Skill



Panel B: High Skill

Figure 4: Distribution of Realized Input Adjustment Costs

This figure shows the distribution (box plot) of the estimated firm-level adjustment costs as a fraction of firms' annual sales (CX/Y) in high- and low-skill industries, using the parameter estimates reported in Table 3, columns (2) and (3), to calculate the adjustment costs. In each box, the central mark is the median, the edges of the box are the 25th and 75th percentiles. The whiskers extend to the most extreme data points the algorithm considers not to be outliers.

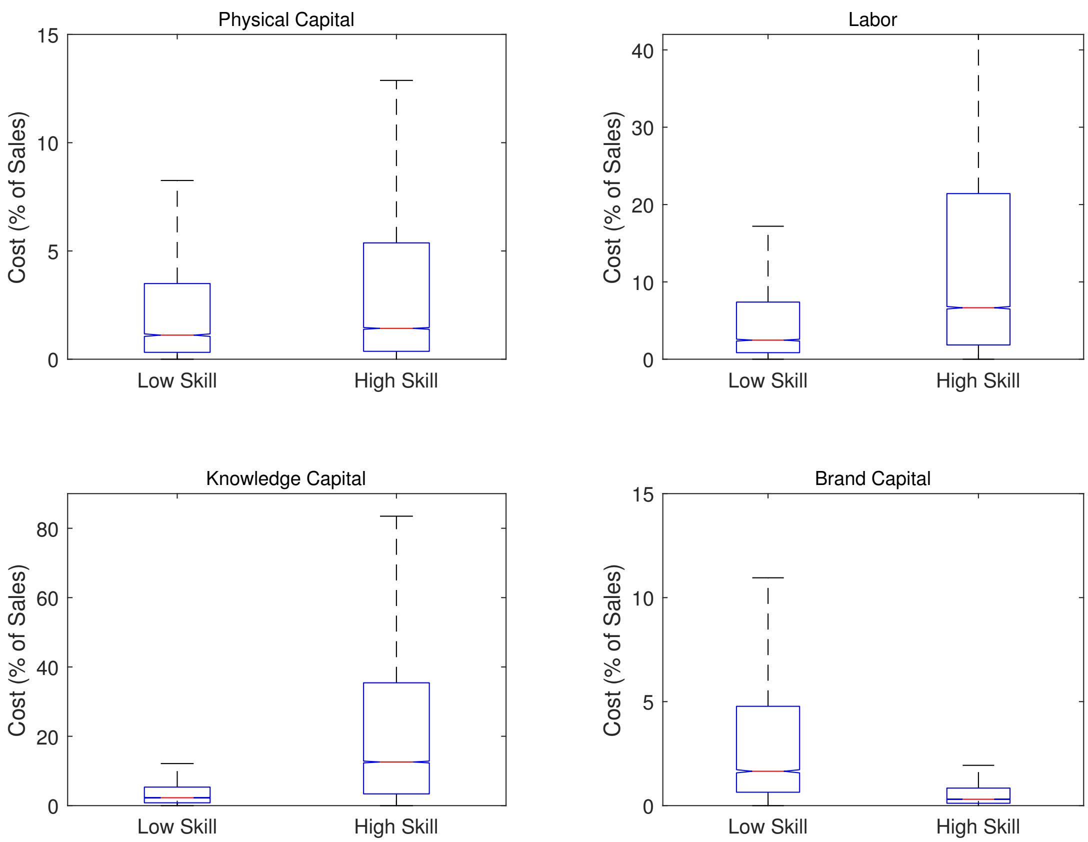
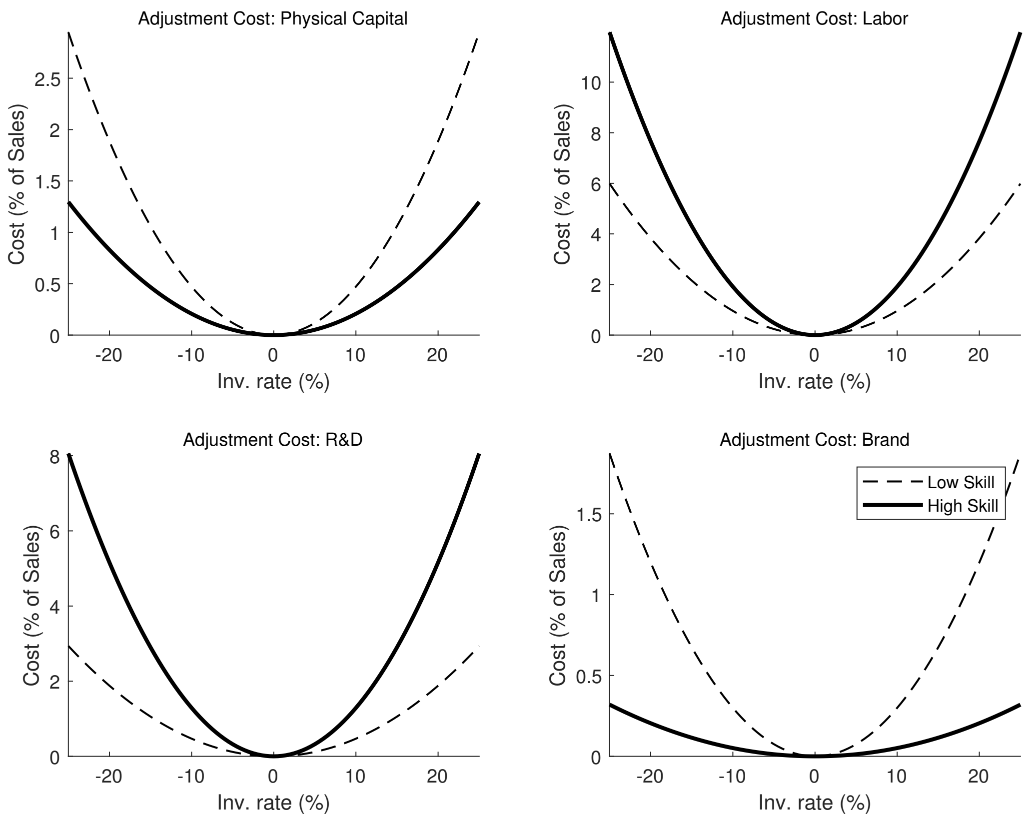


Figure 5: Estimated Adjustment Cost Functions

Panel A in this figure plots the estimated adjustment cost functions for each input in low- and high-skill industries, using the parameter estimates reported in Table 3, columns (2) and (3). The adjustment costs of each input are calculated as a proportion of the respective (average) median input stock-to-sales ratio reported in Table 1. Panel B shows the adjustment costs-to-sales ratio evaluated at the (average) median of corresponding investment rates and as a proportion of the respective (average) median input stock-to-sales ratio reported in Table 1.

Panel A: Adjustment cost functions

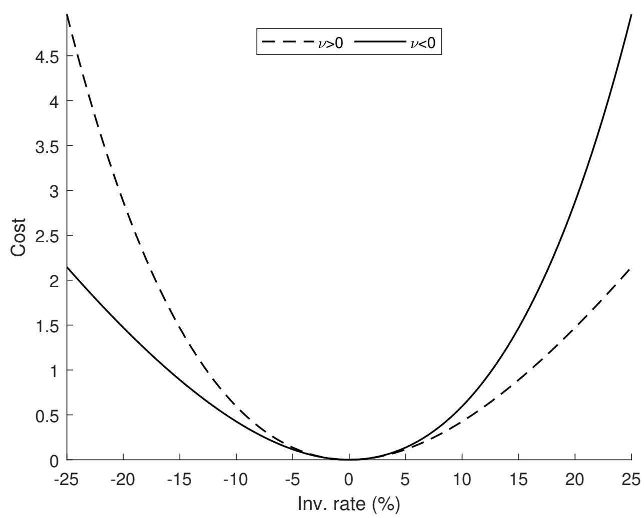


Panel B: Adjustment cost functions evaluated at (average) median investment/hiring rates (in %)

	Low Skill	High Skill
CP/Y : Physical	1.06	1.40
CL/Y : Labor	2.16	5.54
CK/Y : Knowledge	2.07	11.61
CB/Y : Brand	1.73	0.35

Figure 6: Asymmetric Adjustment Costs Function

This figure shows the asymmetric adjustment costs function specification $C = \frac{\theta}{v}\left[\exp\left(-v\frac{I}{K}\right) + v\frac{I}{K} - 1\right]K$, using a slope adjustment cost parameter $\theta = 1$, a capital stock of $K = 1$, with curvatures of $\nu = -5$ (solid) and $\nu = 5$ (dashed). When $v > 0$ it is more costly to desinvest than to invest (to capture irreversibility), and vice versa when $v < 0$.



Appendix

A Derivation of the Firm-Value Decomposition

The first order conditions with respect to I_{it}^P, K_{it+1}^P, H_{it}, L_{it+1}, I_{it}^K, K_{it+1}^K, I_{it}^B, K_{it+1}^B, and B_{it+1}, from maximizing the cum-dividend market value of equity are:

$$q_{it}^P = 1 + (1-\tau_t)\frac{\partial C_{it}}{\partial I_{it}^P} \tag{A.1}$$

$$q_{it}^P = E_t\left[M_{t+1}\left[(1-\tau_{t+1})\left(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}^P} - \frac{\partial C_{it+1}}{\partial K_{it+1}^P}\right) + \delta_{it+1}^P\tau_{t+1} + (1-\delta_{it+1}^P)q_{it+1}^P\right]\right] \tag{A.2}$$

$$q_{it}^L = (1-\tau_t)\frac{\partial C_{it}}{\partial H_{it}} \tag{A.3}$$

$$q_{it}^L = E_t\left[M_{t+1}\left[(1-\tau_{t+1})\left(\frac{\partial \Pi_{it+1}}{\partial L_{it+1}} - \frac{\partial C_{it+1}}{\partial L_{it+1}} - W_{it+1}\right) + (1-\delta_{it+1}^L)q_{it+1}^L\right]\right] \tag{A.4}$$

$$q_{it}^K = (1-\tau_t)\left[1 + \frac{\partial C_{it}}{\partial I_{it}^K}\right] \tag{A.5}$$

$$q_{it}^K = E_t\left[M_{t+1}\left[(1-\tau_{t+1})\left(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}^K} - \frac{\partial C_{it+1}}{\partial K_{it+1}^K}\right) + (1-\delta_{it+1}^K)q_{it+1}^K\right]\right] \tag{A.6}$$

$$q_{it}^B = (1-\tau_t)\left[1 + \frac{\partial C_{it}}{\partial I_{it}^B}\right] \tag{A.7}$$

$$q_{it}^B = E_t\left[M_{t+1}\left[(1-\tau_{t+1})\left(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}^B} - \frac{\partial C_{it+1}}{\partial K_{it+1}^B}\right) + (1-\delta_{it+1}^B)q_{it+1}^B\right]\right] \tag{A.8}$$

$$1 = E_t\left[M_{t+1}\left[r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1}\right]\right] = E_t\left[M_{t+1}r_{it+1}^{Ba}\right]. \tag{A.9}$$

In the last equation we define the after-tax bond return as $r_{it+1}^{Ba} \equiv r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1}$.

Using the FOCs (A.2), (A.4), (A.6), and (A.8) we can write:

$$\begin{aligned}
& q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K K_{it+1}^K + q_{it}^B K_{it+1}^B \\
&= E_t\Bigg[M_{t+1}\Bigg[(1-\tau_{t+1})\left(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}^P}K_{it+1}^P + \frac{\partial \Pi_{it+1}}{\partial L_{it+1}}L_{it+1} + \frac{\partial \Pi_{it+1}}{\partial K_{it+1}^K}K_{it+1}^K + \frac{\partial \Pi_{it+1}}{\partial K_{it+1}^B}K_{it+1}^B\right) \\
&\quad - (1-\tau_{t+1})\left(\frac{\partial C_{it+1}}{\partial K_{it+1}^P}K_{it+1}^P + \frac{\partial C_{it+1}}{\partial L_{it+1}}L_{it+1} + \frac{\partial C_{it+1}}{\partial K_{it+1}^K}K_{it+1}^K + \frac{\partial C_{it+1}}{\partial K_{it+1}^B}K_{it+1}^B\right) \\
&\quad + (1-\delta_{it+1}^P)q_{it+1}^P K_{it+1}^P + (1-\delta_{it+1}^L)q_{it+1}^L L_{it+1} + (1-\delta_{it+1}^K)q_{it+1}^K K_{it+1}^K + (1-\delta_{it+1}^B)q_{it+1}^B K_{it+1}^B \\
&\quad + \delta_{it+1}^P\tau_{t+1}K_{it+1}^P - (1-\tau_{t+1})W_{it+1}L_{it+1}\Bigg]\Bigg].
\end{aligned}$$

Given the homogeneity of degree one of the operating profit function and the adjustment costs function, we have:

$$
\begin{aligned}
& q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K K_{it+1}^K + q_{it}^B K_{it+1}^B \\
= & E_t \left[M_{t+1} \left[(1-\tau_{t+1})(\Pi_{it+1} - C_{it+1} - I_{it+1}^K - I_{it+1}^B - W_{it+1} N_{it+1}) - I_{it+1}^P + \delta_{it+1}^P \tau_{t+1} K_{it+1}^P \right.\right. \\
& + (1-\tau_{t+1}) \frac{\partial C_{it+1}}{\partial I_{it+1}^P} I_{it+1}^P + I_{it+1}^P + (1-\tau_{t+1}) \frac{\partial C_{it+1}}{\partial H_{it+1}} H_{it+1} + (1-\tau_{t+1}) \frac{\partial C_{it+1}}{\partial I_{it+1}^K} I_{it+1}^K \\
& + I_{it+1}^K + (1-\tau_{t+1}) \frac{\partial C_{it+1}}{\partial I_{it+1}^B} I_{it+1}^B + I_{it+1}^B \\
& + (1-\delta_{it+1}^P) q_{it+1}^P K_{it+1}^P + (1-\delta_{it+1}^L) q_{it+1}^L L_{it+1} + (1-\delta_{it+1}^K) q_{it+1}^K K_{it+1}^K \\
& \left.\left. + (1-\delta_{it+1}^B) q_{it+1}^B K_{it+1}^B \ \right]\right] \\
= & E_t \left[M_{t+1} \left[(1-\tau_{t+1})(\Pi_{it+1} - C_{it+1} - I_{it+1}^K - I_{it+1}^B - W_{it+1} N_{it+1}) - I_{it+1}^P + \delta_{it+1}^P \tau_{t+1} K_{it+1}^P + B_{it+2} - r_{it+1}^B B_{it+1} \right.\right. \\
& \left.\left. + q_{it+1}^P K_{it+2}^P + q_{it+1}^L L_{it+2} + q_{it+1}^K K_{it+2}^K + q_{it+1}^B K_{it+2}^B - B_{it+2} \ \right]\right] + E_t \left[M_{t+1} r_{it+1}^B \right] B_{it+1}.
\end{aligned}
$$

Rearranging the above equation,

$$
q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K K_{it+1}^K + q_{it}^B K_{it+1}^B - B_{it+1} = E_t \left[M_{t+1} \left[\begin{array}{c} D_{it+1} + q_{it+1}^P K_{it+2}^P \\ + q_{it+1}^L L_{it+2} + q_{it+1}^K K_{it+2}^K + q_{it+1}^B K_{it+2}^B - B_{it+2} \end{array} \right] \right].
$$

Recursively applying the above the equation to future periods,

$$
\begin{aligned}
& q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K K_{it+1}^K + q_{it}^B K_{it+1}^B - B_{it+1} \\
= & E_t \left[M_{t+1} D_{it+1} + M_{t+2} D_{it+2} + M_{t+2} \left[q_{it+2}^P K_{it+3}^P + q_{it+2}^L L_{it+3} + q_{it+2}^K K_{it+3}^K + q_{it+2}^B K_{it+3}^B - B_{it+3} \right] \right] \\
= & \ldots \\
= & \sum_{\triangle t=1}^{\infty} M_{t+\triangle t} D_{it+\triangle t} + \lim_{\triangle t \to \infty} E_t \left[M_{t+1} \left[q_{it+\triangle t}^P K_{it+\triangle t}^P + q_{it+\triangle t}^L L_{it+\triangle t} + q_{it+\triangle t}^K K_{it+\triangle t}^K + q_{it+\triangle t}^B K_{it+\triangle t}^B - B_{it+\triangle t} \right] \right].
\end{aligned}
$$

Assuming that the transversality condition holds then,

$$
q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K K_{it+1}^K + q_{it}^B K_{it+1}^B = V_{it} - D_{it} + B_{it+1} = P_{it} + B_{it+1}.
$$